Seifedine Kadry · Abdelkhalak El Hami
Editors

Numerical Methods for Reliability and Safety Assessment

Multiscale and Multiphysics Systems

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ISBN 978-3-319-07166-4 ISBN 978-3-319-07167-1 (eBook)
DOI 10.1007/978-3-319-07167-1
Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014948196

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Foreword

The recent advances in materials, sensors, and computational methods have resulted in a much higher reliability and safety expectations of infrastructures, products, and services. This has been translated into expected longer lives for non-repairable products such as satellites, longer warranty periods for both repairable and non-repairable products such as automobiles, and longer residual lives of infrastructures such as bridges, dams, and high-rising buildings. In order to accomplish these expectations, the designers, engineers, and analysts need to incorporate the system configuration, physics of failure of its components, and the scale and complexity of the system. Therefore, testing begins at the components levels and subsystems. Reliability and safety analyses are conducted at all levels considering different failure modes of the components and subsystems under different operating conditions. Different numerical approaches are required at every aspect and step in the design and implementation processes.

The chapters of this book cover three topics related to different aspects of reliability and safety of complex systems. The first set of topics deals with generic methods and approaches which include theoretical developments and quantification of uncertainties which have effects on the expected lives and performance of the products and structures, approaches for risk assessments due to environmental conditions, methods for conducting and analyzing accelerated life testing, and use of advanced design of experiments methods such as Latin Hypercube for estimating the optimum parameters levels for reliability-based designs. The second set of topics deals with applications and use of reliability as a criterion in the design of civil engineering infrastructures such as blast wall structures, road pavements operating under different environmental conditions and different traffic loads, and other applications. The third set of topics is devoted to mechanical systems, their designs and reliability modeling. They include optimum inspection periods for aircraft structures subject to fatigue loadings and optimum repairs for mechatronics systems.

The book is an excellent reference for the design of systems, structures, and products for reliability and safety. The chapters provide coverage of the use of reliability methods in a wide range of engineering applications.

Piscataway, NJ E.A. Elsayed

Preface

Reliability and safety analyses are important applications of modern probabilistic methods and stochastic concept (reliability of systems, probability of failure, statistics, and random variables/processes). These fields create a wide range of problems but due to their practical importance, it gave rise to development of new probabilistic methods and can contain interesting and fruitful mathematical settings. The reliability of a structure is traditionally achieved by deterministic methods using safety factors calculated generally under conservative estimators of influent parameters. Structural reliability analysis methods use probabilistic approaches for assessing safety factors or for optimizing maintenance and inspection programs. These methods become essential in the frame of long-term maintenance or life extension.

The main focus of this book is numerical methods for multiscale and multiphysics in reliability and safety. Multiphysics problems are problems involving two or more equations describing different physical phenomena that are coupled together via the equations. Multiscale problems on the other hand are problems on large scales that experience fine scale behavior, which makes them hard to solve using standard methods. Instead of solving the entire problem at once the problem is rewritten into many smaller subproblems that are coupled from each other.

This book includes 29 chapters, contributed by worldwide researchers and practitioners from 16 countries, of innovative concepts, theories, techniques, and engineering applications in various fields. It is designed to assist practicing engineers, students, and researchers in the areas of reliability engineering, safety and risk analysis.

Egaila, Kuwait — Seifedine Kadry
Rouen, France — Abdelkhalak El Hami

Acknowledgments

We would like to thank the Springer editor and the book reviewers for their valuable suggestions to make this quality book project happen and appear to service the public.

Contents

Part I Reliability Education

Part II Uncertainty Quantification and Uncertainty Propagation Analysis

Part III Reliability and Risk Analysis

Part IV Decision Making Under Uncertainty

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Part I
Reliability Education

Mechanical System Lifetime

Raed Kouta, Sophie Collong, and Daniel Play

Abstract We present, in three parts, the approaches for the random loading analysis in order to complete methods of lifetime calculation.

First part is about the analysis methods. Second part considers modeling of random loadings. A loading, or the combination of several loadings, is known as the leading cause of the dwindling of the mechanical component strength. Third part will deal with the methods taking into account the consequences of a random loading on lifetime of a mechanical component.

The motivations of the present document are based on the observation that operating too many simplifications on a random loading lost much of its content and, therefore, may lose the right information from the actual conditions of use. The analysis of a random loading occurs in several ways and in several approaches, with the aim of later evaluate the uncertain nature of the lifetime of a mechanical component.

Statistical analysis and frequency analysis are two complementary approaches. Statistical analyses have the advantage of leading to probabilistic models (Demoulin B (1990a) Processus aléatoires [R 210]. Base documentaire « Mesures. Généralités ». (*)) provide opportunities for modeling the natural dispersion of studied loadings and their consequences (cracking, fatigue, damage, lifetime, etc.). The disadvantage of these statistical analyses is that they ignore the history of events.

On the other hand, the frequency analyses try to remedy this drawback, using connections between, firstly, the frequencies contained in the loading under consideration and, secondly, whether the measured average amplitudes (studied with

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S. Kadry and A. El Hami (eds.), *Numerical Methods for Reliability and Safety Assessment: Multiscale and Multiphysics Systems*, DOI 10.1007/978-3-319-07167-1_1,
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the Fourier transform, FT) or their dispersions (studied with the power spectral density, PSD) (Kunt M (1981) Traitement numérique des signaux. Éditions Dunod; Demoulin B (1990b) Fonctions aléatoires [R 220]. Base documentaire « Mesures. Généralités ». (*)). The disadvantage of frequency analyses is the need to issue a lot of assumptions and simplifications for use in models of lifetime calculation (e.g., limited to a system with one degree of freedom using probabilistic models simplified for the envelope of the loading).

A combination of the two analyses is possible and allows a good fit between the two approaches. This combination requires a visual interpretation of the appearance frequency. Thus, a random loading is considered a random process to be studied at the level of the amplitude of the signal, its speed, and its acceleration.

1 Random Loadings Analysis

1.1 Usual Conditions of a Mechanical System

Mechanical systems and mechanical components provide functions for action more or less complicated. These actions are performed and controlled by one or more users in a variety of conditions (Schütz 1989). The diversity of uses leads to a large number of load situations. The challenge for designers of mechanical systems and mechanical components integrates these actual conditions of use (Heuler and Klätschke 2005). More generally, the challenge is to take into account the possibly nondeclared or explicit wishes of the users. Practically, it is to consider the diversity of loads and stresses applied to mechanical components. This condition is added to the geometric optimization requirements and conditions of material strength (Pluvinage and Sapunov 2006). It requires the development of a calculation tool suitable for both to obtain a representative model of loads and to carry out design calculations (Weber 1999).

Taking into account the actual conditions of use become a technological and economic challenge. But it causes a profound change in attitude since the causes are considered a probable way from assumptions used by a significant segment of the population. And of course, the calculation of the effects will be presented in terms of probability of strength and reliability (Lannoy 2004). This approach is possible because the two parts of the modeling are now well understood. Firstly, the effects of various loads applied to the components can be analyzed and calculated in terms of dynamic loads (Savard 2004; Bedard 2000). Then, the physical behavior of materials subjected to repeated stress is better known (Lu 2002; Rabbe and Galtier 2000). The design engineer can then develop methods for calculations reconciling best current knowledge and objectives he must achieve. Upstream of the approach, the loads from the conditions imposed by the users must be known. And downstream of the approach, it is necessary to calculate the consequences of such loads.

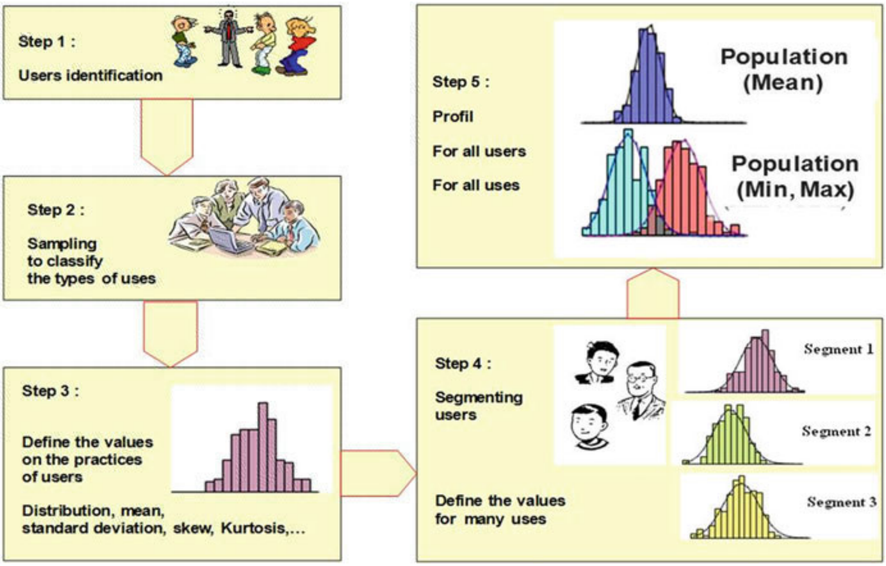

Fig. 1 Taking into account conditions of use

The variety of conditions is the major difficulty encountered in the integration of real condition of use when designing a mechanical component. For example, an equipments model is designed as a response to the needs of a user class (Fig. 1). A user class or class of use (Heuler and Klätschke 2005; Lallet et al. 1995; Leluan 1992a; Ten Have 1989) is often determined based on a profile of life confirmed by a market investigation. Despite the definition of multiple use classes, constructors seek as much as possible on the operating parts of equipments, to make the offer more overall that is to say, to find integrators resemblances between different classes use. The simplest presentation of a class of use or a life profile in the field of transport is by example to determine the number of kilometers traveled by an average user will during a specified period. This number of kilometers is presented as a sum weighted of a set of types of severity often called mission profiles or slices of life (good road, bad road, roundabout, mountain, city, different climatic conditions, etc.). Even if these simplified configurations, predictive calculations of resistance and lifetime require a statement of simplifications and assumptions that lead to the use of safety factors (Clausen et al. 2006) to reduce the risk of defects. Indeed, a class of use (or life profile) is considered by the designer of mechanical components, such as a homogeneous whole. Nevertheless, this homogeneity is accompanied by uncertainties that require consideration in terms of random information. Indeed, it is now proved (Osgood 1982) that a random mechanical stress leads to a lifetime smaller than alternating stress which seems broadly similar.

1.2 Statistical Analysis or Counting Methods of Random Loads

We are interested here in the methods of interpretation of the characteristic parameters of time series (or a discrete graphics representation) to obtain a distribution law of these parameters (Brozzetti and Chabrolin 1986a). From the viewpoint of checking the fatigue of the mechanical components, the extent of variation of the variable load is an essential parameter of the same value as the average stress. Variable loads may come as external actions as internal stresses. In what follows, we shall make no distinction knowing that it is possible to determine the stresses from the variable actions applied to a structure or component, making either quasi-static or dynamic mechanical calculations.

1.2.1 Load Event

The term "load event" (Grubisic 1994) gives rise to a history of stress (also called trajectory). This load event is a load state of service, characteristic of the mechanical system and generating within each component considered, a variation of stimulations.

Examples Included in the transport sector are the following cases:

- Bridge-road: the passage of a vehicle characterized by mass, number of axles, the speed, producing a bias at a particular point of the structure. The passage of the vehicle being a function of several parameters (the surface irregularities of the coating, the transverse position of the vehicle on the item, the weight of the rolling load, speed, etc.).
- Road-chassis: the stresses on the chassis of a vehicle on a road section.
- Marine platform: the action of water depending on the status of storm characterized by the duration, the height, the period, the average direction of waves.

Know the statistical distribution of load events during the intended use of the system or the mechanical component, then leads to the establishment of a statistical distribution law given by the average number of occurrences of each type of event. For a bridge, this distribution is that of the expected traffic; for the chassis of a vehicle, are the driving conditions; and for a marine structure, it will be a weather data on the frequency of storms.

When each load event is characterized by one or more parameters, the long-term distribution is in the form of a histogram, easily representable for one or two parameters (Rabbe et al. 2000a).

In some cases, experience and theoretical modeling used to have this distribution analytically. The histogram obtained is then replaced by a continuous distribution law. The majority of mechanical systems and mechanical components from simple to more complicated are subject to loads distributions often represented by Weibull laws (Chapouille 1980).

For example in the field of land transport, this distribution may relate to severe stresses in a car chassis such as:

$$p\left(C_{\mathrm{s}} > c^*\right) = \exp\left[-\left(\frac{c^*}{c_0}\right)^{\gamma}\right]$$

and

$$p\left(c^* \leq C_{\mathrm{s}} < c^* + dc^*\right) = \frac{\gamma}{c_0}\left(\frac{c^*}{c_0}\right)^{\gamma-1} \exp\left[-\left(\frac{c^*}{c_0}\right)^{\gamma}\right] dc^*$$

$p(C_{\mathrm{s}} > c^*)$ represents the probability of exceeding a threshold c^*; C_{s} is the random variable representing a severe stress event which can be here a stress due to the passage on a road in poor condition and shown in a significant stress c_0 ; $p(c^* \leq C_{\mathrm{s}} < c^* + dc^*)$ represents the probability of being located around a threshold. It is thus possible to assess the probability that this stress is between c^* and $c^* + dc^*$. For this example, the statistical knowledge of the total number of sections of bad road then used to define a number of instances is to be associated with a given state of stress.

The difficulty of estimating a statistical distribution load event is that any statistical prediction as it relates to natural events (wind, wave, current, etc.) or in-service use of a considered mechanical system or considered mechanical component (traffic on a bridge, resistance of a frame, etc.). This prediction on the probability distribution of load events can be challenged by the emergence of exceptional causes.

Example We may not have anticipated increased traffic on a bridge for special seasonal reasons. Similarly another example, it is always difficult to extrapolate over the long term, the extreme value of a wave height, based on statistical values of wave heights measured in a few months.

1.2.2 Load Spectrum (Grubisic 1994), Histogram

Example Acceleration recorded on the axle as it passes on a test section, the speed of a gust of wind during a given period, etc.

From this trajectory, the problem is to obtain the information necessary to have at a histogram, or a distribution law of stresses that is called the spectrum of loads or stresses (Grubisic 1994), which is in reality only approximate representation of all charges stresses applied. We also note that obtaining load spectrum reduced information, in the sense that you lose the timing of the cycles of load variations. Therefore, the subsequent calculation of the damage (presented in the third part) may not consider any interaction between successive cycles of stress variations due to these loads. It may, however, admit that many events are largely random and it is unrealistic, at the stage of predicting the behavior of mechanical systems and components claim to have any knowledge of the precise order of appearance, e.g., values of variations ranges of stresses. It thus focuses on the study of the statistical distribution of variations ranges of stresses. And for some applications, the average stress of each cycle is sometimes used. Assume in the following presentation, the overall average stress is zero on the duration of the path.

Statement of Characteristic Data of a Random Loading

Except for some special cases of process (periodic sinusoidal path, stationary narrow band Gaussian process, that is to say with few excitation characteristic frequencies), it is generally difficult to combine with a variations range of stresses of one cycle (Fig. 2). In the case of a very irregular loading path, such as that of Fig. 2, the secondary peaks are problematic. And any a priori definition of how to count variations ranges of stresses may lead to differences in prediction compared to reality, if it is not supported by experimental verification.

The laws of damage based on more or less simple models (Duprat 1997), and the only way to tell if an identification of damaging cycles method is better than another is to correlate the results with those of the studied model of experience where it is possible to achieve (time scale and/or compatible cost, etc.). In fact, the existing methods give results fairly dispersed compared to published (Chang and Hudson 1981) results. For these reasons, the extraction of information from a random stress is to be performed with care. Different types of information to be extracted may occur in the following three forms:

Global analysis: where all the amplitudes of the stress are considered regardless the geometrical shape of the path (amplitude extreme, positive or negative slope, curvature upwards or downwards). This analysis is done using the histogram of the stress or by tracking specific amplitudes in the stress studied.

Local analysis: through the study of extreme values according to the geometrical shape of the path. In this case, the extreme values are separated into four statistical groups: the positive peaks, the trough positive, peaks negative, and trough negative. Amplitudes that do not have a change of direction in the digitized signal are not studied.

Analyses of stress tracts and/or of cycles: When the random stress is considered a constraint, it is useful to think in tract or stress cycle. This definition is consistent with what is done during fatigue tests under sinusoidal stresses that life is measured by the number of cycles. In the case of a sinusoidal stress, a tract concerns only half a cycle. In the case of random stress $S(t)$, defining a cycle is less easy:

- *Definition of a peak and a trough of stress*

  A stress peak S_M (or a trough stress S_m) is defined as the value of a local maximum (or local minimum) of the function $S(t)$. This peak (or the trough) can be positive or negative.

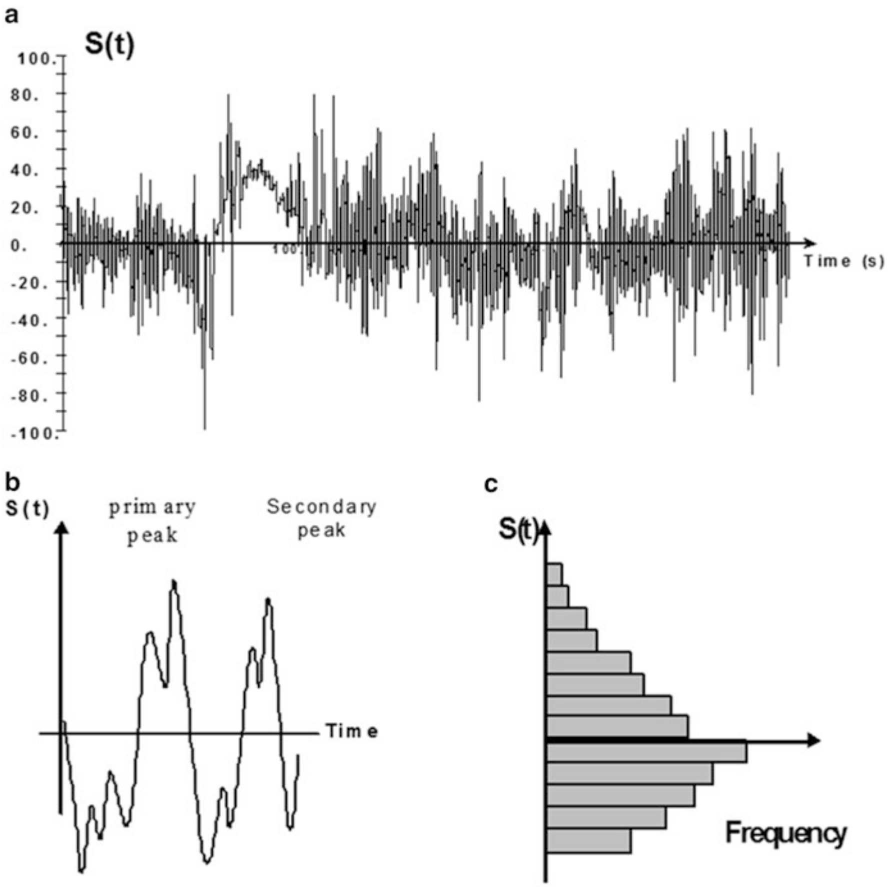

Fig. 2 Viewing a solicitation provided actual use. (**a**) Temporal solicitation, (**b**) detail of signal, (**c**) histogram

- *Definition of a half cycle or one cycle of tract stress variation*

  A tract half-cycle variation of stress is defined by the time between two successive local extreme values of S_{M} and S_{m} (the tract of the variation of stress is defined by $S = S_{\mathrm{M}} - S_{\mathrm{m}}$) (Fig. 3a). A tract cycle of stress variation is defined as the time between two successive local maxima whose value is the first S_{M} and the second is S'_{M} (Fig. 3a), intermediate local minimum with a value S_{m}. The extent of variation of stress associated with this cycle is not unique in this case, since it may be taken as

$$S = |S_{\mathrm{M}} - S_{\mathrm{m}}| \quad \text{or} \quad S' = \left|S'_{\mathrm{M}} - S_{\mathrm{m}}\right|.$$

  Another way to define a cycle, and that this is not linked to the counting of the peaks and troughs of a path, is related to the time interval between two zero crossings and by increasing value (or decreasing values) of the path (Fig. 3b).

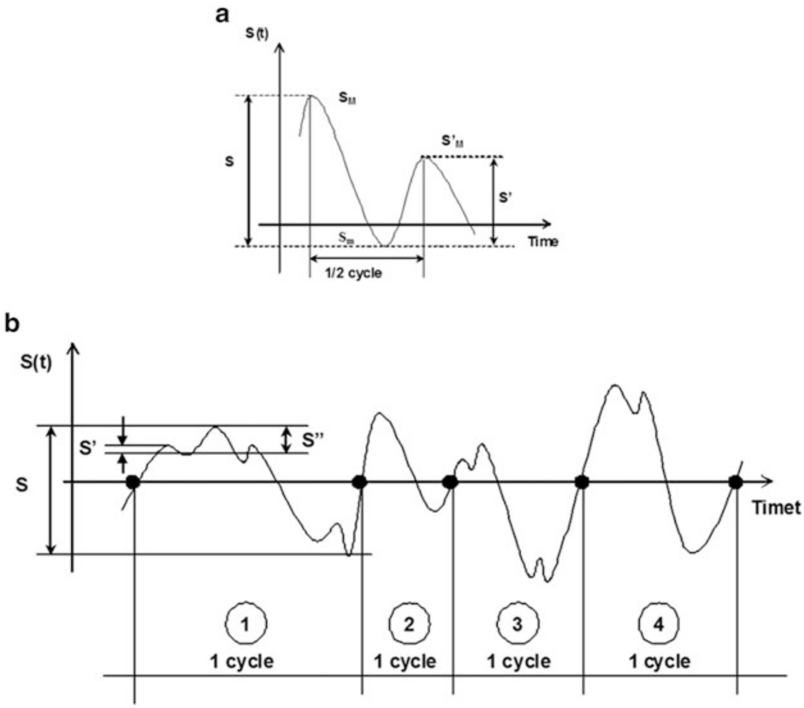

Fig. 3 Definition of characteristics of stress. (**a**) Half cycle definition, (**b**) series of cycles

The example of Fig. 3b with local peaks and local troughs shows the difficulty in defining one cycle and the tract of variation of stress associated with this cycle. Only the cycle no. 2 in the figure is used to define a single tract of variation of stress associated with this cycle.

In summary, three pieces of information must be seen from a random loading: the amplitudes that have imposed load considered (overall analysis), specific amplitudes observed by zooming effect and that reflect the severity of loading (local analysis), and finally tract or extracts cycles of loads studied. And counting methods (Lalanne 1999a) can be divided into three groups: global methods, local methods, and the methods of counting matrix.

Counting Global Methods

The main global counting methods (Lalanne 1999a) are: counting by class and method count overruns levels. For each counting method, an application will be presented around the stress shown in Fig. 4a.

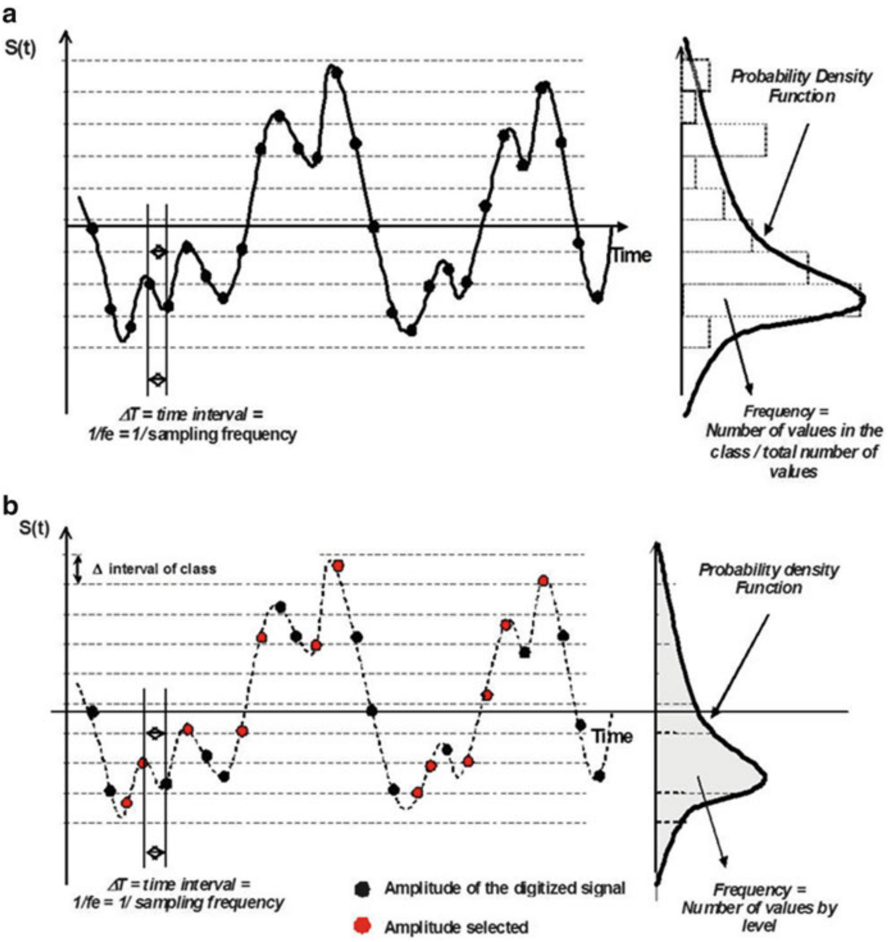

Fig. 4 Descriptive statistics of stress. (**a**) Realization of histogram, (**b**) definition of probability density function

Histogram or Holding Time in a Class of Amplitudes

This method considers the digital signal recorded as a statistical sample not knowing the temporal aspect. The sample is grouped into classes of amplitudes (Fig. 4a, dashed horizontal lines). In this case, no distinction is made between the extreme values and others. The advantages of this method reside in the immediate possibility of statistical modeling and propose a model of probability density (right side of Fig. 4a). Since between two successive points, there is a not predefined time by the method of measurement, the number of points recorded in a class when multiplied by the time step gives the total holding time of the stress studied in this class of amplitudes. This counting method should be reserved only for homogeneous stress (or whose source is considered homogeneous) that is to say, if no significant change in the nature of loading. Indeed, this method is very dependent on the speed (first derivative of the curve considered on a path with a specific dynamic signature) and the acceleration (second derivative) of the stress studied. In the case where the stress has several types of information (related to the braking, cornering, various loads, etc.), the signal loses its homogeneity and the counting class will be altered by these class different uses which dynamic signature is not the same. Figure 4a shows a digitized stress where 28 points and 9 classes of amplitudes are defined. The counting class (from the class below) leads successively to 2, 7, 3, 5, 1, 2, 5, 1, and 2 amplitudes per class.

Counting the Number of Level Crossing

This method, like the previous one, calls for predefined amplitude classes (Fig. 4b). Counting, for a given level, is triggered when the signal exceeds a level with a positive slope (hence the name level crossing). A count of the number of given level crossing is only relevant if an attitude selection of small oscillations is defined. These small oscillations can provide loads staffing (number) without interest from the point of view of calculating the damage and calculating life. And for counting the number of level crossing increment signal noted Δ is defined. It is often in the case of a mechanical component, interval stress below a fatigue limit set to a Wöhler curve (Leybold and Neumann 1963). This increment is considered Δ a threshold reset in the counting process. Historically, several counting methods have been proposed. The most interesting method has only one level if the stress has already gone through at least once this threshold Δ, irrespective of nature of slope. It should also be remembered that the counting is done with a digitized signal, and a proximity rule should be implemented to count very close to the levels determined amplitude. This counting method allows—as the previous one—to build a model of probability density. The application of this method focuses on the levels defined by the class boundaries. Count per level (with the solicitation of Fig. 4b, from low level) leads successively to 0, 0, 2, 1, 0, 1, 1, and 2 level overruns.

Counting Local Methods (Local Events)

Extreme values (peaks and troughs) of a random stress occur from four different families by:

- Positive maximum values preceded by a positive slope (peak > 0)
- Negative minimum values preceded by a negative slope (trough < 0)
- Negative maximum values preceded by a positive slope (peak < 0)
- Positive minimum values preceded by a negative slope (trough > 0)

Figure 5 illustrates these four families of amplitudes. As for the global analysis, grouping into classes for each family gives the possibility to build a model of probability density by type of extreme values. The result of this counting method around the stress shown in Fig. 5 leads, starting from the low class of the nine amplitude classes, to the results in Table 1.