Undergraduate Lecture Notes in Physics

Kurt Fischer

Relativity for Everyone

How Space-Time Bends

Second Edition

Springer

Undergraduate Lecture Notes in Physics

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To Yukiko
You edited the text to become readable

Preface

Dear Reader,

When I was a teenager, I first encountered the theory of relativity in the public library of my hometown. There were two kinds of books in this subject. The easy ones did not really explain but displayed large colored pictures showing some kind of science fiction. The serious books seemed to explain things that nevertheless remained hidden behind a mess of mathematical symbols and as a result they did not *really* explain. So I was back to square one. Nevertheless, it aroused my interest in physics. It also motivated me to fill the gap. The result is this book.

This book is about light, energy, mass, space, time, and gravity: it is through these concepts that we explain the theory of special relativity and the theory of gravity, known as the **theory of general relativity**.

We will use many **thought experiments** and show how physicists create and solve models. This method is the one used by Einstein himself. He understood the theory through both *physical* and *geometrical* pictures. We will follow Einstein's original train of thoughts as closely as possible, even using some of his own thought experiments.

We will present some involved arguments. Nevertheless you only need your imagination but no complicated mathematics to understand the essence of it all. However, the beauty of physics is that we *can* calculate the numbers. Therefore, we present the equations of the theory of relativity together with the most important *exact* solutions only by using elementary mathematics. Even the Einstein equation of gravity showing the bending of space and time, we present and solve *in detail* and in common language. By the end, we will see why the theory of general relativity is the *simplest theory* of gravity.

That is to say there is a common misconception about general relativity: that it is impossible to understand without higher mathematics, and that it is therefore only for a few experts. However, already in 1973 the famous textbook "Gravitation"[1] advises on how to explain general relativity.

> Only three basic principles are invoked: special relativity physics, the equivalence principle, and the local nature of physics. They are simple and clear. To apply them, however, imposes a double task: (A) take space-time apart into locally flat pieces (where the principles are valid), and (B) put these pieces together again into a comprehensible picture. To undertake this dissection and reconstitution, to see curved dynamic space-time inescapably take form, and to see the consequences for physics: that is general relativity.

I am convinced, and believe that you will be too, that the book you hold in your hands realizes this concept.

Synopsis of the Contents

In the first four chapters, we explain what is called the theory of special relativity. We describe the relation between light, matter, space, and time.

1. In Chap. 1, we introduce the basics and describe that mass and energy are the opposite side of the same coin.
2. In Chap. 2, we will see why time and length are "relative."
3. In Chap. 3, we will see why any current-carrying wire exhibits relativity in everyday life.
4. In Chap. 4, we learn that, while riding a merry-go-round, school geometry is no longer true.

In Chaps. 5–9, we explain the theory of general relativity which describes gravity.

5. In Chap. 5, we show that Earth's gravity does not pull at all, but rather that it bends space *and* time.
6. In Chap. 6, we will see in detail the effects of the bending of space and time.
7. In Chap. 7, we thoroughly explain the meaning of the famous Einstein equation of gravity and the reason why it is the simplest possible way of describing gravity.
8. In Chap. 8, we introduce the most famous *exact* solution of the Einstein equation, i.e., the Schwarzschild solution, in simple terms.
9. In Chap. 9, we use the solutions of the Einstein equation and explore the famous predictions of the theory of general relativity, e.g., how much a light beam bends while passing at the sun, how large and heavy black holes are, why the orbits of the planets revolve slowly around the sun, and why the universe had a Big Bang but why its future is unclear to us.

[1] C.W. Misner, K.S. Thorne, and J.A. Wheeler: *Gravitation*, Freeman (1973)

Units and Symbols

One remark on highlighted words: indexed words appear in **boldface.** This means you can more easily find them on the corresponding pages.

Before describing the strange properties of light, we first explain units for measuring and compare the size of things. In physics, we use certain **units** to measure things. We measure lengths only in meters, time only in seconds, *not* in minutes, hours, or days. We measure mass only in kilograms. Any *other* unit used in this book is a combination of these units. For example, speed is measured in meters per second. Other units such as pounds or inches or the like are never used. The merit of this is that we can drop the units in all calculations because we know anyway which units are to be added *afterward* and we *fixed* them at the beginning.

We will often encounter really large or very small numbers. For example, while numbers like "one thousand" can be written down as 1000, the number one billion two hundred fifty-two million seven hundred eighty-four (1,250,000,784) is much harder to read. Usually, we are only interested in the first three digits or so for a rough estimate of how large things are. Here physicists count the number of digits after the first one—that is nine in this case—and they write

$$1.25 \times 10^9$$

In the same way, we write a very small number like 0.000145 as 1.45×10^{-4} by counting the number of leading zeros. Then we can easily multiply such numbers: we multiply $1.25 \times 10^9 \times 1.45 \times 10^{-4}$ by first multiplying 1.25 and 1.45 which is roughly 1.81, and adding the exponents $9 + (-4) = 5$. The result is $\approx 1.81 \times 10^5$. Here the symbol $\approx$ means "**is roughly equal to**".

For example, for the speed of light we usually use the rough value

$$\text{speed of light} \approx 3.00 \times 10^8 \text{ meters per second} \tag{1}$$

We collect other important numbers appearing throughout the text in Table A.1 for reference.

Special Wording

And lastly, one remark about how we use phrases like "far enough away from something," "fast enough," and the like. For example, we say

> if the astronaut is far enough away from Earth, the astronaut can nearly neglect Earth's gravity.

We are aware of the fact that gravity is *never* exactly zero, even *very* far away from Earth. The fact of the matter is that the astronaut wants to measure some effect without gravity spoiling the measurement by the one percent of the result that the astronaut would get in a *really* empty space. So, if the astronaut thinks that gravity is still spoiling his experiment too much, he is *free to move* to a place which is *so far away* from Earth that there gravity *does* really spoil his experiment only up to the desired one percent at the most. Of course, if he wants to measure even more accurately, he must move even farther away from Earth. In short, if he moves "far enough away" from Earth, he can always neglect Earth's gravity *to the extent that he wants to* neglect it.

Tokuyama, Japan
April 2015

Kurt Fischer

Contents

Chapter 1
Light, Matter, and Energy

1.1 Light Beams

An astronaut floating in space switches on a torch or a laser beamer. The emerging light beam travels at the speed

$$299,792,458 \text{ meters per second} \tag{1.1}$$

This is the **speed of light in vacuum**. *Exactly*.

How can we verify this? First of all, we need an apparatus to send a light beam such as the box on the left in Fig. 1.1. It is open to the right. This box symbolizes the torch, or the laser, or the *sender* for short. The black horizontal arrow stands for the light beam. It travels through the gray box on the right which is some speedometer *measuring* the speed of the light beam in meters per second. We will *not* discuss what constitutes such an apparatus: we just assume that there *are* such devices.

1.2 First Law of Relativity: Straight, Steady Speed Is Relative

Does the speed of light change if we move the torch while sending a light beam? This prompts the question: *is moving relative* to what?

If we are moving in a fast train, we do *not* feel the steady speed of the train but only feel a slight tremble: that is the *non-steady* part of the speed. For example, in Fig. 1.2 a table stands inside a train. The train is moving straightly and steadily. The black ball will not begin to move on the table. Do you feel the tremendous speed while sitting at a table at home and feeling to be at rest? Which speed? For example, we could be referring to the tremendous speed at which the Earth is moving around the sun at least during a few minutes, say nearly straight and steady, or the speed at

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Fig. 1.1 Light beam from *left to right,* sketched as *black horizontal arrow*. The *box* on the *left* stands for some sender like a torch or a laser; the *gray box* on the *right* is the light-speedometer, displaying the speed of light

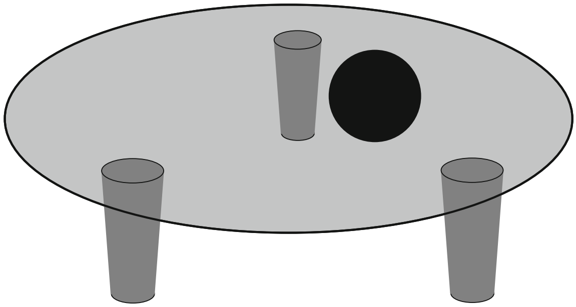

Fig. 1.2 The ball on the glass table in a train will not begin to move on the table, if the train moves straightly and steadily

which the whole solar system is moving along the galaxy and/or the speed at which the galaxy is moving—to where?

Since Galilei at the end of the Middle Ages, we know that we *cannot* detect the speed if we are steadily moving straight ahead in some direction, by *any means whatsoever*. We can always take the point of view that *we are at rest* in the same way as we do when we "sit at a table". Even while sitting at table in a train that is steadily moving straight ahead, we can say that *we* are at rest and the whole station, plus the surrounding area we pass through, is moving *towards* us. At the same time, a friend standing on the platform of the station objects: of course the train is moving towards the station and the station together with the Earth around it are at rest!

What is correct? Answer: *both* we and our friend are right about insisting to be at rest: the train is moving only *relative* to the train station and the Earth around it. This is the **first law of the theory of relativity** formulated by **Galilei** a few hundred years ago:

> The speed of straight, steady motion of a body can only be measured *relative* to other bodies. The laws of nature do *not* depend on a straight and steady speed at which we may move relative to other bodies.