

SPRINGER BRIEFS IN PHYSICS

Rodolfo Figari
Alessandro Teta

Quantum Dynamics of a Particle in a Tracking Chamber



Springer

SpringerBriefs in Physics

Editorial Board

Egor Babaev, University of Massachusetts, USA

Malcolm Bremer, University of Bristol, UK

Xavier Calmet, University of Sussex, UK

Francesca Di Lodovico, Queen Mary University of London, UK

Maarten Hoogerland, University of Auckland, New Zealand

Eric Le Ru, Victoria University of Wellington, New Zealand

Hans-Joachim Lewerenz, California Institute of Technology, USA

James Overduin, Towson University, USA

Vesselin Petkov, Concordia University, Canada

Charles H.-T. Wang, University of Aberdeen, UK

Andrew Whitaker, Queen's University Belfast, UK

For further volumes:

<http://www.springer.com/series/8902>

Rodolfo Figari · Alessandro Teta

Quantum Dynamics of a Particle in a Tracking Chamber

 Springer

Rodolfo Figari
Dipartimento di Fisica - INFN
sezione di Napoli
Università Federico II
Napoli
Italy

Alessandro Teta
Dipartimento di Matematica
Sapienza Università di Roma
Roma
Italy

ISSN 2191-5423 ISSN 2191-5431 (electronic)
ISBN 978-3-642-40915-8 ISBN 978-3-642-40916-5 (eBook)
DOI 10.1007/978-3-642-40916-5
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013949183

© The Author(s) 2014

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law. The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

A tracking chamber is a device in which atoms or sub-atomic particles are detected and their dynamical life (generally very short by the standards of our everyday life) is recorded.

The classical description of what happens in the chamber is very well established and sounds entirely conceivable: the particle, emitted, for example, by a radioactive source, interacts with the microscopic constituents of a large environment kept in an unstable state. Local modifications of the environment, triggered by many small interactions with the moving particle, are enhanced by the long-range nonlinear interactions among the environment constituents to become the macroscopic footprints of the particle passage.

The observed tracks of the particle passage appear, in fact, as straight or curved lines which are properly described as trajectories of a classical particle in a classical electromagnetic field. For a theoretical description of the process the quantum character of the particle dynamics must be taken into account. In particular, it is relevant that the initial state of the particle emitted by the source does not have the form of a semi-classical wave packet (i.e., well localized in position and momentum) but rather that it is in a highly correlated superposition state, e.g., a spherical wave. Thus arises the nontrivial problem of explaining how such a superposition state can produce the observed classical trajectories.

In the standard approach, based on the so-called “orthodox” interpretation of quantum mechanics, a possible explanation is given considering the environment as a classical object that causes the “collapse” of the wave function of the particle.

In other words, the environment is interpreted as being a “measurement apparatus” that gives rise to the transition from the quantum behavior of the particle to the classical observed behavior of the particle in the chamber.

Nevertheless, a different point of view is worth considering. In fact, since the early days of quantum mechanics, the description of the observed dynamics of a particle in a tracking chamber (the Wilson cloud chamber in those days) generated an intense debate. Two main features made the case of particular interest: (i) there is no straightforward interpretation of the cloud chamber’s supersaturated vapor as a measurement apparatus: what does it measure? This leaves unspecified consequences and timing of the wave function collapse and, (ii) in principle the process of ionization of the vapor atoms might be dealt with in pure quantum mechanical terms.

Taking these aspects into consideration, another approach has been proposed in which the observed tracks ought to be explained as the outcome of the entire system Schrödinger's dynamics (of the particle and of the environment), instead of being deduced to be the consequence of the wave function reduction axiom.

In 1929, Mott (1929) examined such a possibility. In spite of its undeniable interest, Mott's detailed perturbative analysis remained almost unnoticed until recently.

Since the 1960s, there have been new investigations into the border between classical and quantum, measured system and measuring apparatus, starting with the theoretical work of Bell (1987) and with a few accurate experimental investigations. These works motivated the first attempts at modeling quantum measuring apparatuses and thermal baths. Of particular relevance to the field was the study of decoherence induced in quantum systems by interaction with an environment and the consequent emergence of a classical behavior in quantum systems (Joos et al. 2003).

The case of a particle in a tracking chamber is a prototypical example of classical behavior induced by the environment which, in our opinion, has received insufficient attention in the literature. A more detailed examination of this case study, based on a rigorous analysis of quantum models of the whole system, could be of great significance for a more profound comprehension of the conceptual structure of quantum mechanics and of its connection with the classical world.

This monograph provides a historical introduction to the problem of particle tracks in a cloud chamber and a quantitative account of recent attempts to describe a particle in a tracking chamber.

In [Chap. 1](#) we provide a brief summary of some basic facts about the original Wilson chamber and describe a few early attempts to explain the observed tracks based on quantum mechanics. We then give a detailed account of Mott's 1929 paper to emphasize its role in pioneering the investigation into the dynamics of a quantum particle in a quantum environment. We conclude the chapter with an outline of successive studies on some related topics.

[Chapter 2](#) is devoted to a detailed reformulation of Mott's three-particle model in a completely time-dependent setting. In order to have a simpler description of the forces confining electrons to the atoms of the chamber we use point interaction potentials. A different model-atom of the quantum environment is successively analyzed where the electron is harmonically bound to a force center in a fixed position.

In [Chap. 3](#) we consider a quantum environment made of localized spins or, much the same thing, of two level atoms. We will use, as a technical tool in the construction of such an environment, the theory of multi-channel point interactions.

Our primary aim in [Chaps. 2](#) and [3](#) is to present perturbative and non-perturbative strategies to characterize qualitatively and quantitatively the Schrödinger dynamics of a quantum particle in an array of model-atoms.

For simplicity, in some cases proofs of the main results are only outlined. The reader is referred to the original papers for more details.

In **Chap. 4** we collect some concluding remarks where we also list few open problems we consider particularly relevant.

In the appendices we give a survey of the standard point interaction theory and we recall some elementary facts about the spherical wave.

We thank G. Dell'Antonio and S. Albeverio for helpful and stimulating discussions on the subject of this work and also on many other research topics during years of intense scientific collaboration and friendship. We also thank C. Cacciapuoti, R. Carlone, D. Finco, D. Noja, and C. Recchia for the careful reading of part of the manuscript and several useful suggestions.

Rodolfo Figari
Alessandro Teta

References

- Bell, J.: Speakable and unspeakable in quantum mechanics. Cambridge University Press,(1987)
Joos, E., Zeh, H. D., Kiefer, C., Giulini, D., Kupsch, J., Stamatescu, I. O.: Decoherence and the Appearance of a Classical World in Quantum Theory. 2nd ed., Springer, Berlin (2003)
Mott, N.F.: The wave mechanics of α -ray tracks. Proc. R. Soc. Lond. A **126**, 79–84 (1929)

Contents

1	Tracks in the Cloud Chamber	1
1.1	The Wilson Cloud Chamber	1
1.2	The Birth of Quantum Mechanics	2
1.3	The Debate at the Fifth Solvay Conference	6
1.4	Heisenberg’s Chicago Lectures	8
1.5	Towards a Purely Wave-Mechanical Approach	12
1.6	Mott’s Analysis	13
1.7	Successive Investigations	20
1.7.1	Models of Quantum Measurement Devices	21
1.7.2	Decoherence Induced by Scattering	24
	References	28
2	A Time-Dependent Analysis of Mott’s Model	31
2.1	The Three-Particle Model Revisited	31
2.2	Time-Dependent Formulation and Results	36
2.3	Representation Formulas by Oscillatory Integrals	39
2.4	Proof of Mott’s Result	42
2.5	Asymptotic Dynamics in Presence of One Model-Atom	52
2.6	Outline of the Proof	57
	References	61
3	Modeling Atoms as Multi-Channel Point Interactions	63
3.1	Multi-Channel Point Interactions	63
3.2	A Model of Cloud Chamber	69
3.3	The Case of a Macroscopic Array of Spins	76
3.4	The Spherical Wave Amplitudes	81
	References	86
4	Concluding Remarks	87
	References	90
	Appendix A: Point Interaction Hamiltonians	91
	Appendix B: The Spherical Wave	99

Chapter 1

Tracks in the Cloud Chamber

This chapter is devoted to a historical introduction of the problem of tracks observed in a cloud chamber. Continuing along the lines of the subject we presented in our paper (Figari and Teta 2013), we briefly describe the experimental apparatus, revisit the main ideas leading to the formulation of quantum theory in the years 1925–1927 and analyze the first attempts at a quantum theoretical explanation of observed tracks presented by Born, Heisenberg and Darwin. We then give a detailed account of N.F. Mott’s 1929 paper, which clearly outlines a pioneering approach to the dynamics of a quantum particle interacting with a quantum environment. We conclude by briefly recalling some subsequent developments about two topics that we consider are closely related to our main subject: models of quantum measurement devices and the theory of scattering induced decoherence.

1.1 The Wilson Cloud Chamber

The cloud chamber that C.T.R. Wilson constructed during the years 1911–1912 is the prototype for what today would be called a tracking chamber (see e.g., Leone and Robotti (2004) for a description of the original apparatus). The relevance of the Wilson chamber was immediately acknowledged: Ernest Rutherford described the cloud chamber as *the most original and wonderful instrument in scientific history*. The device was used extensively to explore the properties of many different atomic and sub-atomic particles. In particular, it was used to investigate the “ionizing radiation” emitted by radioactive sources.

During the final years of the nineteenth century, Wilson had been conducting experiments on the phenomenon of fog formation in air saturated with water vapor, examining the role of electric charges as condensation nuclei for the excess of vapor. As he pointed out in his Nobel Prize lecture in December 1927, during the first years of the twentieth century *...ideas on the corpuscular nature of alpha-and beta-rays had become much more definite, and I had in view the possibility that the track of an*

ionizing particle might be made visible and photographed by condensing water on the ions which it liberated.

The experimental apparatus he made available operates schematically as follows. The air contained in the chamber, saturated with water vapor, is brought into a super-saturated state by means of a fast expansion that suddenly lowers its temperature. The α -particle, released by a radioactive source in the center of the chamber, interacts with the atoms of the gas, inducing ionization. The ionized atoms then act as condensation centers, giving rise to the formation of small drops of water. A photograph of the visible track made by the sequence of these drops is the extraordinary output of the experiment. Synchronizing the expansion, illumination and photography was one of the complex experimental problems Wilson had to cope with.

The observed tracks are interpreted as magnifications of the α -particle “trajectories”. In fact, they are accurately characterized as trajectories of a charged classical particle (relativistic or non-relativistic according to the initial particle velocity) in a classical electromagnetic field. And, in particular, they are straight lines whenever no electromagnetic field is present.

1.2 The Birth of Quantum Mechanics

It is well known that the development of ideas leading to the final formulation of quantum mechanics took place over a long period of time, going back to the first work on black body radiation law developed by M. Planck in 1900 ((Jammer 1989; see also Cushing 1994) for a critical historical analysis of some interpretational problems).

In our brief historical survey here, we will limit ourselves to recalling just a few important contributions made during the crucial years of 1925–1927 that we consider decisive in the elaboration of the new theory. Our aim is simply to provide some historical context for where matters stood with regard to the theory when the quantum mechanical description of a particle in a cloud chamber was first attempted.

It is widely accepted that the pivotal contributions to the elaboration of the theory came from Werner Heisenberg, Max Born and Pascual Jordan in 1925 (Heisenberg 1925; Born and Jordan 1925) and from Erwin Schrödinger in 1926 (Schrödinger 1978). In spite of their radically different basic strategies, the two approaches, known as matrix mechanics and wave mechanics, respectively, were immediately recognized as mathematically equivalent theories by Schrödinger himself.

Heisenberg’s primary idea was that classical kinematics, based on position and velocity of point particles, is inadequate to describe microscopic particle motion (e.g., electrons in an atom). The main reason for this conclusion is that position and velocity are quantities that can barely be observed at atomic level, and a proper physical theory should always rely on observable quantities. Furthermore, there was, according to Heisenberg, a stronger reason to abandon classical kinematics. Within the framework of the old quantum theory, it was assumed that an electron could only occupy one of the admissible stationary states labeled by integers. The transition between two stationary states (quantum jump) was associated with the emission or

adsorption of a quantum of light, with frequency given by the Einstein-Bohr rule. Such a description already introduces an evident discontinuity in the motion of the electron that was clearly incompatible with classical kinematics.

This then led Heisenberg to propose a new kinematics based entirely on observable quantities which, at the atomic level, should be identified with frequencies and amplitudes of the radiation emitted or adsorbed in quantum jumps. He noticed that these quantities are characterized by two indices (specifying the initial and the final stationary states) and that their product is not commutative. As for the dynamics, the assumption was that two rules of the old quantum theory (Newton's law and the selection rule for stationary states) should remain valid and that they had only to be conveniently rewritten in terms of the new observable quantities.

In 1925 Born and Jordan finalized Heisenberg's program. Using a Hamiltonian formalism, they were able to associate with the observable quantities introduced by Heisenberg infinite, hermitian matrices obeying the canonical commutation relations. This led to the first formulation of matrix mechanics that appeared to describe atomic phenomena satisfactorily. It is worth emphasizing that the theory relied on the explicit rejection of the idea of a continuous motion in ordinary space to describe the dynamical evolution of a microscopic object. It thus represented a radical departure from the classical conception of the description of a physical process.

Schrödinger elaborated a different approach. Following previous hypotheses put forward by Louis de Broglie, Schrödinger pursued the idea that an electron must be described as a wave rather than as a point particle. The main motivation behind this was that the wave equation, with suitable boundary conditions, admits standing wave solutions corresponding to a discrete set of possible frequencies. This opened the possibility of obtaining, via the Einstein-Bohr rule, the discrete energy levels that seemed to be a typical feature of any atomic system. In his search for such an equation, Schrödinger used the formal analogy between classical mechanics and geometrical optics and the fact that geometrical optics is recovered as the short wavelength limit of wave optics. Guided by this analogy, he succeeded in formulating the equation for the new wave mechanics, which came to be known as the Schrödinger equation. One of the main features of the theory was that the equation reproduced, in the short wavelength limit, the laws of ordinary classical mechanics of point particles.

In Schrödinger's view the wave function $\psi(x, t)$, the solution of his evolution equation, had a concrete physical meaning. In particular, he proposed considering the electron charge distributed in space with a charge density that was given by $e|\psi(x, t)|^2$, at time t (here e is the electron charge). The conservation of the total charge was then guaranteed by the conservation of the L^2 -norm of the solutions of the equation and by the choice of a normalized wave function at time zero.

Schrödinger expected that an electron moving in space would be represented by solutions in the form of wave packets remaining well concentrated during time evolution. Schrödinger himself verified the existence of solutions of this kind in the special case of the harmonic oscillator. However, a serious difficulty arose as soon as it was realized that, except in the special case of the harmonic oscillator, the solutions would inevitably spread in space as time goes by.