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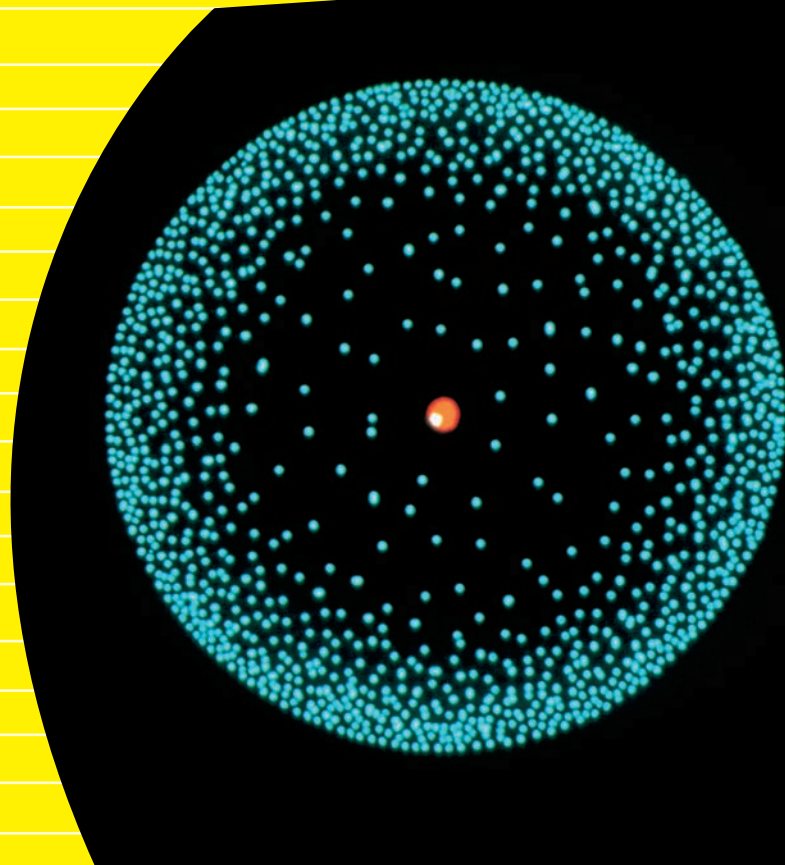
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Steven Holzner, PhD

Author, Differential Equations For Dummies



Differential Equations Workbook

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Differential Equations Workbook

FOR
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by Steven Holzner, PhD



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Steven Holzner is the award-winning author of many books, including *Differential Equations For Dummies* and *Physics For Dummies*. He did his undergraduate work at MIT and got his PhD at Cornell University. He's been on the faculty of both MIT and Cornell.

Dedication

To Nancy, of course.

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Introduction

Too often, differential equations seem like torture. They seem so bad in fact that you may be tempted to cringe or shudder when you're assigned homework that involves 'em.

Differential Equations Workbook For Dummies may not get you to embrace differential equations with open arms, but it *will* improve your understanding of the pesky things. Here you get ample practice working through the most common types of differential equations, along with detailed solutions, so you can truly master the subject. Get ready to add "differential equations expert" to your résumé!

About This Book

Differential Equations Workbook For Dummies is all about practicing solving differential equations. It's crammed full of the good stuff — and *only* the good stuff. Each aspect of differential equations is addressed with some brief text to refresh your memory of the basics, a worked-out example, and multiple practice problems. (If you're looking for in-depth explanation of differential equations topics, your best resource is *Differential Equations For Dummies* [Wiley] or your class textbook.) So that you're not left hanging wondering whether your solution is right or wrong, each chapter features an answers section with all the practice problems worked out, step by glorious step.

You can leaf through this workbook as you like, solving problems and reading solutions as you go. Like other *For Dummies* books, this one is designed to let you skip around to your heart's content.

Conventions Used in This Book

Some books have a dozen confusing conventions that you need to know before you can even start. Not this one. You need to keep just these few things in mind:

- ✓ New terms appear in *italics* the first time they're presented. And like other math books, this one also employs italics to indicate variables.
- ✓ Web sites appear in `mono font` to help them stand out. (In some cases, a Web site may break across multiple lines. Rest assured I haven't inserted any extra spaces or punctuation; just type the address as provided.)
- ✓ In the answers section at the end of every chapter, the practice problems and solutions appear in **bold** (the step-by-step info that follows is in regular text). Matrices and key-words in bulleted lists are also given in bold.

Foolish Assumptions

Any study of differential equations takes knowledge of calculus as its starting point. You should know how to take basic derivatives and how to integrate before reading this workbook (and if you don't, I recommend picking up a copy of *Calculus For Dummies* [Wiley] first).

Most importantly, I'm assuming you already have an in-depth resource about differential equations available to you. This workbook is intended to give you extra practice tackling standard differential equations concepts; it doesn't provide detailed instruction on the fundamentals of differential equations. I do include some brief refresher text on each aspect of differential equations, but if you're brand-new to the subject, check out *Differential Equations For Dummies* or your class textbook.

How This Book Is Organized

This workbook is organized modularly, into parts, following the same organizational structure as *Differential Equations For Dummies*. Here's what you're going to find in each part.

Part I: Tackling First Order Differential Equations

First order differential equations are the easiest differential equations to solve. That's why this part gives you practice finding solutions to linear, separable, and exact first order differential equations.

Part II: Finding Solutions to Second and Higher Order Differential Equations

The most interesting differential equations used in the real world are second order differential equations. Here, you practice multiple ways of solving this type of equation. You also get to try your hand at solving third and higher order differential equations. Things get pretty steep pretty fast, but fortunately you have some surprising techniques at your disposal, as you discover in this part.

Part III: The Power Stuff: Advanced Techniques

I've pulled out all the stops in Part III. In these chapters, you find some powerful solution techniques, including power series, which you can use to convert a tough differential equation into an algebra problem and then solve for the coefficients of each power, and Laplace transforms, which can occasionally give you the solution you're looking for in no time.

Part IV: The Part of Tens

The classic *For Dummies* Part of Tens provides you with a couple collections of top ten resources. Flip to this part to find help with the ten common ways of solving differential equations or to discover ten real-world applications for differential equations.

Icons Used in This Book

For Dummies books always use icons to point out important information; this workbook is no different. Here's the quick-and-dirty of what the icons mean:



This icon points out practice problems that have been worked out for you to get you off on the right foot.



Looking for the juicy tidbits that are essential to your study of differential equations? Then watch for paragraphs marked with this icon.



This icon denotes tricks and techniques to make your life easier (at least as it relates to solving differential equations).

Where to Go from Here

You can start anywhere you feel you need the most practice. In fact, this workbook was written to allow you to do just that. However, if you want to follow along with *Differential Equations For Dummies* or your textbook, your best bet is to start with Chapter 1.

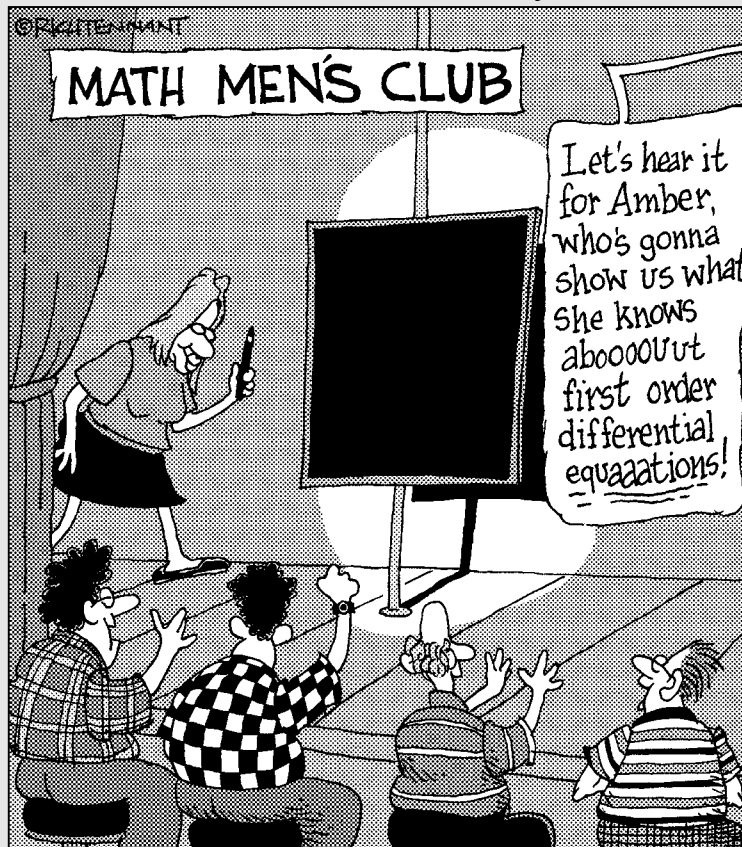
You may also want to grab a few pieces of scratch paper. I've tried to leave you enough room to work the problems right in the book, but you still might find a little extra paper helpful.

Part I

Tackling First Order Differential Equations

The 5th Wave

By Rich Tennant



In this part . . .

Welcome to the world of first order differential equations! Here, you put your skills to the test with linear first order differential equations, which means you're dealing with first order derivatives that are to the first power, not the second or any other higher power. You also work with separable first order differential equations, which can be separated so that only terms in y appear on one side of the equation and only terms in x appear on the other side (okay, okay, constants can appear on this side too). Finally, you practice solving exact differential equations.

Chapter 1

Looking Closely at Linear First Order Differential Equations

In This Chapter

- ▶ Knowing what a first order linear differential equation looks like
- ▶ Finding solutions to first order differential equations with and without y terms
- ▶ Employing the trick of integrating factors

One important way that you can classify differential equations is as linear or nonlinear. A differential equation is considered *linear* if it involves only *linear terms* (that is, terms to the power 1) of y , y' , y'' , and so on. The following equation is an example of a linear differential equation:

$$L \frac{d^2 Q}{dx^2} + R \frac{dQ}{dx} + \frac{1Q}{C} = E(x)$$

Nonlinear differential equations simply include nonlinear terms in y , y' , y'' , and so on. This next equation, which describes the angle of a pendulum, is considered a nonlinear differential equation because it involves the term $\sin \theta$ (not just θ):

$$\frac{d^2 \theta}{dx^2} + \frac{g}{L} \sin \theta = 0$$

This chapter focuses on linear first order differential equations. Here you have the chance to sharpen your linear-equation-spotting eye. You also get to practice solving linear first order differential equations when y is and isn't involved. Finally, I clue you in to a little (yet extremely useful!) trick o' the trade called integrating factors.

Identifying Linear First Order Differential Equations



Here's the general form of a linear differential equation, where $p(x)$ and $q(x)$ are functions (which can just be constants):

$$\frac{dy}{dx} + p(x)y = q(x)$$

Following are some examples of linear differential equations:

$$\frac{dy}{dx} = 5$$

$$\frac{dy}{dx} = y + 1$$

$$\frac{dy}{dx} = 3y + 1$$

For a little practice, try to figure out whether each of the following equations is linear or nonlinear.

**Q.**

Is this equation a linear first order differential equation?

$$\frac{dy}{dx} = 17y + 4$$

A. Yes.

This equation is a linear first order differential equation because it involves solely first order terms in y and y' .

-
- 1.** Is this equation a linear first order differential equation?

$$\frac{dy}{dx} = 9y + 1$$

Solve It

- 2.** Is the following a linear first order differential equation?

$$\frac{dy}{dx} = 17y^3 + 4$$

Solve It

3. Is this equation a linear first order differential equation?

$$\frac{dy}{dx} = y \cos(x)$$

Solve It

4. Is the following a linear first order differential equation?

$$\frac{dy}{dx} = x \cos(y)$$

Solve It

Solving Linear First Order Differential Equations That Don't Involve Terms in y



The simplest type of linear first order differential equation doesn't have a term in y at all; instead, it involves just the first derivative of y , y' , y'' , and so on. These differential equations are simple to solve because the first derivatives are easy to integrate. Here's the general form of such equations (note that $q(x)$ is a function, which may be a constant):

$$\frac{dy}{dx} = q(x)$$

Take a look at this linear first order differential equation:

$$\frac{dy}{dx} = 3$$

Note that there's no term in just y . So how do you solve this kind of equation? Just move the dx over to the right:

$$dy = 3dx$$

Then integrate to get

$$y = 3x + c$$

where c is a constant of integration.

To figure out what c is, simply take a look at the initial conditions. For example, say that $y(0)$ — that is, the value of y when $x = 0$ — is equal to

$$y(0) = 15$$

Plugging $y(0) = 15$ into $y = 3x + c$ gives you

$$y(0) = c = 15$$

So $c = 15$ and $y = 3x + 15$. That's the complete solution!

To deal with constants of integration like c , look for the specified initial conditions. For example, the problem you just solved is usually presented as

$$\frac{dy}{dx} = 3$$

where

$$y(0) = 15$$

Time for a more advanced problem! (Note that this one still doesn't involve any simple terms in y .)

$$\frac{dy}{dx} = x^3 - 3x^2 + x$$

where

$$y(0) = 3$$

Because this equation doesn't involve any terms in y , you can move the dx to the right, like this:

$$dy = x^3 dx - 3x^2 dx + x dx$$

Then just integrate to get

$$y = \frac{x^4}{4} - x^3 + \frac{x^2}{2} + c$$

To evaluate c , use the initial condition, which is

$$y(0) = 3$$

Plugging $x = 0 \rightarrow y = 3$ into the equation for y gives you

$$y(0) = 3 = c$$

So the full solution is

$$y = \frac{x^4}{4} - x^3 + \frac{x^2}{2} + 3$$



As you can see, the way to deal with linear first order differential equations that don't involve a term in just y is simply to

1. Move the dx to the right and integrate.
2. Apply the initial conditions to solve for the constant of integration.

Following are some practice problems to make sure you have the hang of it.



Q. Solve for y in this differential equation:

$$\frac{dy}{dx} = 2x$$

where

$$y(0) = 3$$

A. $y = x^2 + 3$

1. Multiply both sides by dx :

$$dy = 2x \, dx$$

2. Integrate both sides to get the following, where c is a constant of integration:

$$y = x^2 + c$$

3. Apply the initial condition to get

$$c = 3$$

4. Having solved for c , you can find the solution to the differential equation:

$$y = x^2 + 3$$

5. Solve for y in this differential equation:

$$\frac{dy}{dx} = 8x$$

where

$$y(0) = 4$$

Solve It

6. What's y in the following equation?

$$\frac{dy}{dx} = 2x + 2$$

where

$$y(0) = 2$$

Solve It

7. Solve for y in this differential equation:

$$\frac{dy}{dx} = 6x + 5$$

where

$$y(0) = 10$$

Solve It

8. What's y in the following equation?

$$\frac{dy}{dx} = 8x + 3$$

where

$$y(0) = 12$$

Solve It

Solving Linear First Order Differential Equations That Involve Terms in y

Wondering what to do if a differential equation you're facing involves both x and y ?

$$\frac{dy}{dx} + p(x)y = q(x)$$

Start by taking a look at this representative problem:

$$\frac{dy}{dx} = ay - b$$

The preceding is a linear first order differential equation that contains both dy/dx and y . How do you handle it and find a solution? By using some algebra, you can rewrite this equation as

$$\frac{dy/dx}{y - (b/a)} = a$$

Multiplying both sides by dx gives you

$$\frac{dy}{y - (b/a)} = a \, dx$$

Congrats! You've just separated x on one side of this differential equation and y on the other, making the integration much easier. Speaking of integration, integrating both sides gives you

$$\ln |y - (b/a)| = ax + C$$

where C is a constant of integration. Raising both sides to the power e gives you this, where c is a constant defined by $c = e^C$:

$$y = (b/a) + ce^{ax}$$

Anything beyond this level of difficulty must be approached in another way, and you deal with such equations throughout the rest of the book.

If you think you have solving linear first order differential equations in terms of y all figured out, try your hand at these practice questions.



Q. Solve for y in this differential equation:

$$\frac{dy}{dx} = 2y - 4$$

where

$$y(0) = 3$$

A. $y = 2 + e^{2x}$

1. Use algebra to get

$$\frac{dy/dx}{y - 2} = 2$$

2. Then multiply both sides by dx :

$$\frac{dy}{y - 2} = 2dx$$

3. Integrate to get

$$\ln |y - 2| = 2x + C$$

4. Then raise e to the power of both sides:

$$y = 2 + e^C e^{2x} = 2 + ce^{2x}$$

5. Finally, apply the initial condition to get

$$y = 2 + e^{2x}$$

9. What's y in the following equation?

$$\frac{dy}{dx} = 4y - 8$$

where

$$y(0) = 5$$

Solve It

10. Solve for y in this differential equation:

$$\frac{dy}{dx} = 3y - 9$$

where

$$y(0) = 9$$

Solve It

-
11. What's y in the following equation?

$$\frac{dy}{dx} = 9y - 18$$

where

$$y(0) = 5$$

Solve It

12. Solve for y in this differential equation:

$$\frac{dy}{dx} = 4y - 20$$

where

$$y(0) = 16$$

Solve It