A History of Mathematics

THIRD EDITION

Uta C. Merzbach and Carl B. Boyer

Foreword by Isaac Asimov

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In memory of Carl B. Boyer
(1906–1976)
—U.C.M.

To the memory of my parents,
Howard Franklin Boyer and
Rebecca Catherine (Eisenhart) Boyer
—C.B.B.

Contents

Foreword to the Second Edition

By Isaac Asimov

Mathematics is a unique aspect of human thought, and its history differs in essence from all other histories.

As time goes on, nearly every field of human endeavor is marked by changes which can be considered as correction and/or extension. Thus, the changes in the evolving history of political and military events are always chaotic; there is no way to predict the rise of a Genghis Khan, for example, or the consequences of the short-lived Mongol Empire. Other changes are a matter of fashion and subjective opinion. The cave-paintings of 25,000 years ago are generally considered great art, and while art has continuously—even chaotically—changed in the subsequent millennia, there are elements of greatness in all the fashions. Similarly, each society considers its own ways natural and rational, and finds the ways of other societies to be odd, laughable, or repulsive.

But only among the sciences is there true progress; only there is the record one of continuous advance toward ever greater heights.

And yet, among most branches of science, the process of progress is one of both correction and extension. Aristotle, one of the greatest minds ever to contemplate physical laws, was quite wrong in his views on falling bodies and had to be corrected by Galileo in the 1590s. Galen, the greatest of ancient physicians, was not allowed to study human cadavers and was quite wrong in his anatomical and physiological conclusions. He had to be corrected by Vesalius in 1543 and Harvey in 1628. Even Newton, the greatest of all scientists, was wrong in his view of the nature of light, of the achromaticity of lenses, and missed the existence of spectral lines. His masterpiece, the laws of motion and the theory of universal gravitation, had to be modified by Einstein in 1916.

Now we can see what makes mathematics unique. Only in mathematics is there no significant correction—only extension. Once the Greeks had developed the deductive method, they were correct in what they did, correct for all time. Euclid was incomplete and his work has been extended enormously, but it has not had to be corrected. His theorems are, every one of them, valid to this day.

Ptolemy may have developed an erroneous picture of the planetary system, but the system of trigonometry he worked out to help him with his calculations remains correct forever.

Each great mathematician adds to what came previously, but nothing needs to be uprooted. Consequently, when we read a book like *A History of Mathematics*, we get the picture of a mounting structure, ever taller and broader and more beautiful and magnificent and with a foundation, moreover, that is as untainted and as functional now as it was when Thales worked out the first geometrical theorems nearly 26 centuries ago.

Nothing pertaining to humanity becomes us so well as mathematics. There, and only there, do we touch the human mind at its peak.

Preface to the Third Edition

During the two decades since the appearance of the second edition of this work, there have been substantial changes in the course of mathematics and the treatment of its history. Within mathematics, outstanding results were achieved by a merging of techniques and concepts from previously distinct areas of specialization. The history of mathematics continued to grow quantitatively, as noted in the preface to the second edition; but here, too, there were substantial studies that overcame the polemics of "internal" versus "external" history and combined a fresh approach to the mathematics of the original texts with the appropriate linguistic, sociological, and economic tools of the historian.

In this third edition I have striven again to adhere to Boyer's approach to the history of mathematics. Although the revision this time includes the entire work, changes have more to do with emphasis than original content, the obvious exception being the inclusion of new findings since the appearance of the first edition. For example, the reader will find greater stress placed on the fact that we deal with such a small number of sources from antiquity; this is one of the reasons for condensing three previous chapters dealing with the Hellenic period into one. On the other hand, the chapter dealing with China and India has been split, as content demands. There is greater emphasis on the recurring interplay between pure and applied mathematics as exemplified in chapter 14. Some reorganization is due to an attempt to underline the impact of institutional and personal transmission of ideas; this has affected most of the pre-nineteenth-century chapters. The chapters dealing with the nineteenth century have been altered the least, as I had made substantial changes for some of this material in the second edition. The twentieth-century material has been doubled, and a new final chapter deals with recent trends, including solutions of some longstanding problems and the effect of computers on the nature of proofs.

It is always pleasant to acknowledge those known to us for having had an impact on our work. I am most grateful to Shirley Surrette Duffy for responding judiciously to numerous requests for stylistic advice, even at times when there were more immediate priorities. Peggy Aldrich Kidwell replied with unfailing precision to my inquiry concerning certain photographs in the National Museum of American History. Jeanne LaDuke cheerfully and promptly answered my appeals for help, especially in confirming sources. Judy and Paul Green may not realize that a casual conversation last year led me to rethink some recent material. I have derived special pleasure and knowledge from several recent publications, among them *Klopfer* 2009 and, in a more leisurely fashion, *Szpiro* 2007. Great thanks are due to the editors and production team of John Wiley & Sons who worked with me to make this edition possible: Stephen Power, the senior editor, was unfailingly generous and diplomatic in his counsel; the editorial assistant, Ellen Wright, facilitated my progress through the major steps of manuscript creation; the senior production manager, Marcia Samuels, provided me with clear and concise instructions, warnings, and examples; senior production editors Kimberly Monroe-Hill and John Simko and the copyeditor, Patricia Waldygo, subjected the manuscript to painstakingly meticulous scrutiny. The professionalism of all concerned provides a special kind of encouragement in troubled times.

I should like to pay tribute to two scholars whose influence on others should not be forgotten. The Renaissance historian Marjorie N. Boyer (Mrs. Carl B. Boyer) graciously and knowledgeably complimented a young researcher at the beginning of her career on a talk presented at a Leibniz conference in 1966. The brief conversation with a total stranger did much to influence me in pondering the choice between mathematics and its history.

More recently, the late historian of mathematics Wilbur Knorr set a significant example to a generation of young scholars by refusing to accept the notion that ancient authors had been studied definitively by others. Setting aside the "*magister dixit*," he showed us the wealth of knowledge that emerges from seeking out the texts.

—Uta C. Merzbach
March 2010

Preface to the Second Edition

This edition brings to a new generation and a broader spectrum of readers a book that became a standard for its subject after its initial appearance in 1968. The years since then have been years of renewed interest and vigorous activity in the history of mathematics. This has been demonstrated by the appearance of numerous new publications dealing with topics in the field, by an increase in the number of courses on the history of mathematics, and by a steady growth over the years in the number of popular books devoted to the subject. Lately, growing interest in the history of mathematics has been reflected in other branches of the popular press and in the electronic media. Boyer's contribution to the history of mathematics has left its mark on all of these endeavors.

When one of the editors of John Wiley & Sons first approached me concerning a revision of Boyer's standard work, we quickly agreed that textual modifications should be kept to a minimum and that the changes and additions should be made to conform as much as possible to Boyer's original approach. Accordingly, the first twenty-two chapters have been left virtually unchanged. The chapters dealing with the nineteenth century have been revised; the last chapter has been expanded and split into two. Throughout, an attempt has been made to retain a consistent approach within the volume and to adhere to Boyer's stated aim of giving stronger emphasis on historical elements than is customary in similar works.

The references and general bibliography have been substantially revised. Since this work is aimed at English-speaking readers, many of whom are unable to utilize Boyer's foreign-language chapter references, these have been replaced by recent works in English. Readers are urged to consult the General Bibliography as well, however. Immediately following the chapter references at the end of the book, it contains additional works and further bibliographic references, with less regard to language. The introduction to that bibliography provides some overall guidance for further pleasurable reading and for solving problems.

The initial revision, which appeared two years ago, was designed for classroom use. The exercises found there, and in the original edition, have been dropped in this edition, which is aimed at readers outside the lecture room. Users of this book interested in supplementary exercises are referred to the suggestions in the General Bibliography.

I express my gratitude to Judith V. Grabiner and Albert Lewis for numerous helpful criticisms and suggestions. I am pleased to acknowledge the fine cooperation and assistance of several members of the Wiley editorial staff. I owe immeasurable thanks to Virginia Beets for lending her vision at a critical stage in the preparation of this manuscript. Finally, thanks are due to numerous colleagues and students who have shared their thoughts about the first edition with me. I hope they will find beneficial results in this revision.

—Uta C. Merzbach
Georgetown, Texas
March 1991

Preface to the First Edition

Numerous histories of mathematics have appeared during this century, many of them in the English language. Some are very recent, such as J. F. Scott's *A History of Mathematics*[1]; a new entry in the field, therefore, should have characteristics not already present in the available books. Actually, few of the histories at hand are textbooks, at least not in the American sense of the word, and Scott's *History* is not one of them. It appeared, therefore, that there was room for a new book—one that would meet more satisfactorily my own preferences and possibly those of others.

The two-volume *History of Mathematics* by David Eugene Smith[2] was indeed written "for the purpose of supplying teachers and students with a usable textbook on the history of elementary mathematics," but it covers too wide an area on too low a mathematical level for most modern college courses, and it is lacking in problems of varied types. Florian Cajori's *History of Mathematics*[3] still is a very helpful reference work; but it is not adapted to classroom use, nor is E. T. Bell's admirable *The Development of Mathematics.*[4] The most successful and appropriate textbook today appears to be Howard Eves, *An Introduction to the History of Mathematics*,[5] which I have used with considerable satisfaction in at least a dozen classes since it first appeared in 1953.

[1]London: Taylor and Francis, 1958.
[2]Boston: Ginn and Company, 1923–1925.
[3]New York: Macmillan, 1931, 2nd edition.
[4]New York: McGraw-Hill, 1945, 2nd edition.
[5]New York: Holt, Rinehart and Winston, 1964, revised edition.

I have occasionally departed from the arrangement of topics in the book in striving toward a heightened sense of historicalmindedness and have supplemented the material by further reference to the contributions of the eighteenth and nineteenth centuries especially by the use of D. J. Struik, *A Concise History of Mathematics.*[6]

The reader of this book, whether layman, student, or teacher of a course in the history of mathematics, will find that the level of mathematical background that is presupposed is approximately that of a college junior or senior, but the material can be perused profitably also by readers with either stronger or weaker mathematical preparation. Each chapter ends with a set of exercises that are graded roughly into three categories. Essay questions that are intended to indicate the reader's ability to organize and put into his own words the material discussed in the chapter are listed first. Then follow relatively easy exercises that require the proofs of some of the theorems mentioned in the chapter or their application to varied situations. Finally, there are a few starred exercises, which are either more difficult or require specialized methods that may not be familiar to all students or all readers. The exercises do not in any way form part of the general exposition and can be disregarded by the reader without loss of continuity.

Here and there in the text are references to footnotes, generally bibliographical, and following each chapter there is a list of suggested readings. Included are some references to the vast periodical literature in the field, for it is not too early for students at this level to be introduced to the wealth of material available in good libraries. Smaller college libraries may not be able to provide all of these sources, but it is well for a student to be aware of the larger realms of scholarship beyond the confines of his own campus. There are references also to works in foreign languages, despite the fact that some students, hopefully not many, may be unable to read any of these. Besides providing important additional sources for those who have a reading knowledge of a foreign language, the inclusion of references in other languages may help to break down the linguistic provincialism which, ostrichlike, takes refuge in the mistaken impression that everything worthwhile appeared in, or has been translated into, the English language.

The present work differs from the most successful presently available textbook in a stricter adherence to the chronological arrangement and a stronger emphasis on historical elements. There is always the temptation in a class in history of mathematics to assume that the fundamental purpose of the course is to teach mathematics. A departure from mathematical standards is then a mortal sin, whereas an error in history is venial. I have striven to avoid such an attitude, and the purpose of the

[6]New York: Dover Publications, 1967, 3rd edition.

book is to present the history of mathematics with fidelity, not only to mathematical structure and exactitude, but also to historical perspective and detail. It would be folly, in a book of this scope, to expect that every date, as well as every decimal point, is correct. It is hoped, however, that such inadvertencies as may survive beyond the stage of page proof will not do violence to the sense of history, broadly understood, or to a sound view of mathematical concepts. It cannot be too strongly emphasized that this single volume in no way purports to present the history of mathematics in its entirety. Such an enterprise would call for the concerted effort of a team, similar to that which produced the fourth volume of Cantor's *Vorlesungen iiber Geschichte der Mathematik* in 1908 and brought the story down to 1799. In a work of modest scope the author must exercise judgment in the selection of the materials to be included, reluctantly restraining the temptation to cite the work of every productive mathematician; it will be an exceptional reader who will not note here what he regards as unconscionable omissions. In particular, the last chapter attempts merely to point out a few of the salient characteristics of the twentieth century. In the field of the history of mathematics perhaps nothing is more to be desired than that there should appear a latter-day Felix Klein who would complete for our century the type of project Klein essayed for the nineteenth century, but did not live to finish.

A published work is to some extent like an iceberg, for what is visible constitutes only a small fraction of the whole. No book appears until the author has lavished time on it unstintingly and unless he has received encouragement and support from others too numerous to be named individually. Indebtedness in my case begins with the many eager students to whom I have taught the history of mathematics, primarily at Brooklyn College, but also at Yeshiva University, the University of Michigan, the University of California (Berkeley), and the University of Kansas. At the University of Michigan, chiefly through the encouragement of Professor Phillip S. Jones, and at Brooklyn College through the assistance of Dean Walter H. Mais and Professors Samuel Borofsky and James Singer, I have on occasion enjoyed a reduction in teaching load in order to work on the manuscript of this book. Friends and colleagues in the field of the history of mathematics, including Professor Dirk J. Struik of the Massachusetts Institute of Technology, Professor Kenneth O. May at the University of Toronto, Professor Howard Eves of the University of Maine, and Professor Morris Kline at New York University, have made many helpful suggestions in the preparation of the book, and these have been greatly appreciated. Materials in the books and articles of others have been expropriated freely, with little acknowledgment beyond a cold bibliographical reference, and I take this opportunity to express to these authors my warmest gratitude. Libraries and publishers have been very helpful in providing information and

illustrations needed in the text; in particular it has been a pleasure to have worked with the staff of John Wiley & Sons. The typing of the final copy, as well as of much of the difficult preliminary manuscript, was done cheerfully and with painstaking care by Mrs. Hazel Stanley of Lawrence, Kansas. Finally, I must express deep gratitude to a very understanding wife. Dr. Marjorie N. Boyer, for her patience in tolerating disruptions occasioned by the development of yet another book within the family.

—Carl B. Boyer
Brooklyn, New York
January 1968

1

Traces

Did you bring me a man who cannot number his fingers?
—From the *Egyptian Book of the Dead*

Concepts and Relationships

Contemporary mathematicians formulate statements about abstract concepts that are subject to verification by proof. For centuries, mathematics was considered to be the science of numbers, magnitudes, and forms. For that reason, those who seek early examples of mathematical activity will point to archaeological remnants that reflect human awareness of operations on numbers, counting, or "geometric" patterns and shapes. Even when these vestiges reflect mathematical activity, they rarely evidence much historical significance. They may be interesting when they show that peoples in different parts of the world conducted certain actions dealing with concepts that have been considered mathematical. For such an action to assume historical significance, however, we look for relationships that indicate this action was known to another individual or group that engaged in a related action. Once such a connection has been established, the door is open to more specifically historical studies, such as those dealing with transmission, tradition, and conceptual change.

Mathematical vestiges are often found in the domain of nonliterate cultures, making the evaluation of their significance even more complex. Rules of operation may exist as part of an oral tradition, often in musical or verse form, or they may be clad in the language of magic or ritual. Sometimes they are found in observations of animal behavior, removing them even further from the realm of the historian. While studies of canine arithmetic or avian geometry belong to the zoologist, of the impact of brain lesions on number sense to the neurologist, and of numerical healing incantations to the anthropologist, all of these studies may prove to be useful to the historian of mathematics without being an overt part of that history.

At first, the notions of number, magnitude, and form may have been related to contrasts rather than likenesses—the difference between one wolf and many, the inequality in size of a minnow and a whale, the unlikeness of the roundness of the moon and the straightness of a pine tree. Gradually, there may have arisen, out of the welter of chaotic experiences, the realization that there are samenesses, and from this awareness of similarities in number and form both science and mathematics were born. The differences themselves seem to point to likenesses, for the contrast between one wolf and many, between one sheep and a herd, between one tree and a forest suggests that one wolf, one sheep, and one tree have something in common—their uniqueness. In the same way it would be noticed that certain other groups, such as pairs, can be put into one-to-one correspondence. The hands can be matched against the feet, the eyes, the ears, or the nostrils. This recognition of an abstract property that certain groups hold in common, and that we call "number," represents a long step toward modern mathematics. It is unlikely to have been the discovery of any one individual or any single tribe; it was more probably a gradual awareness that may have developed as early in man's cultural development as the use of fire, possibly some 300,000 years ago.

That the development of the number concept was a long and gradual process is suggested by the fact that some languages, including Greek, have preserved in their grammar a tripartite distinction between 1 and 2 and more than 2, whereas most languages today make only the dual distinction in "number" between singular and plural. Evidently, our very early ancestors at first counted only to 2, and any set beyond this level was designated as "many." Even today, many people still count objects by arranging them into sets of two each.

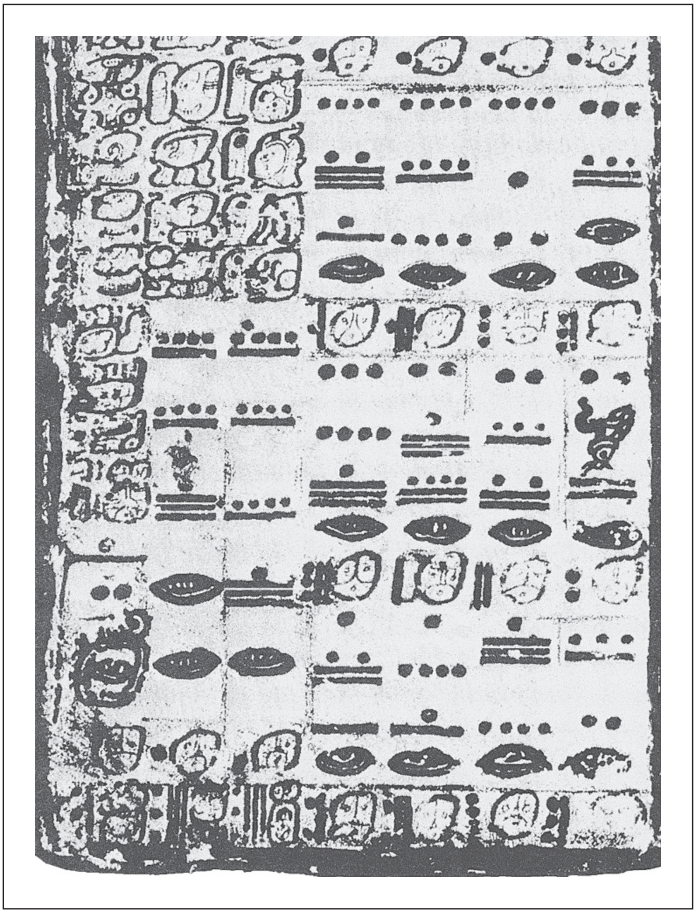
The awareness of number ultimately became sufficiently extended and vivid so that a need was felt to express the property in some way, presumably at first in sign language only. The fingers on a hand can be readily used to indicate a set of two or three or four or five objects, the number 1 generally not being recognized at first as a true "number." By the use of the fingers on both hands, collections containing up to ten elements could be represented; by combining fingers and toes, one could count as high as 20. When the human digits were inadequate, heaps of stones or knotted strings could be used to represent a correspondence with the elements of another set. Where nonliterate peoples used such a scheme of representation, they often piled the stones in groups of five, for they had become familiar with quintuples through observation of the human hand and foot. As Aristotle noted long ago, the widespread use today of the decimal system is but the result of the anatomical accident that most of us are born with ten fingers and ten toes.

Groups of stones are too ephemeral for the preservation of information; hence, prehistoric man sometimes made a number record by cutting notches in a stick or a piece of bone. Few of these records remain today, but in Moravia a bone from a young wolf was found that is deeply incised with fifty-five notches. These are arranged in two series, with twenty-five in the first and thirty in the second: within each series, the notches are arranged in groups of five. It has been dated as being approximately 30,000 years old. Two other prehistoric numerical artifacts were found in Africa: a baboon fibula having twenty-nine notches, dated as being circa 35,000 years old, and the Ishango bone, with its apparent examples of multiplicative entries, initially dated as approximately 8,000 years old but now estimated to be as much as 30,000 years old as well. Such archaeological discoveries provide evidence that the idea of number is far older than previously acknowledged.

Early Number Bases

Historically, finger counting, or the practice of counting by fives and tens, seems to have come later than counter-casting by twos and threes, yet the quinary and decimal systems almost invariably displaced the binary and ternary schemes. A study of several hundred tribes among the American Indians, for example, showed that almost one-third used a decimal base, and about another third had adopted a quinary or a quinary-decimal system; fewer than a third had a binary scheme, and those using a ternary system constituted less than 1 percent of the group. The vigesimal system, with the number 20 as a base, occurred in about 10 percent of the tribes.

An interesting example of a vigesimal system is that used by the Maya of Yucatan and Central America. This was deciphered some time before the rest of the Maya languages could be translated. In their representation of time intervals between dates in their calendar, the Maya used a place value numeration, generally with 20 as the primary base and with 5 as an auxiliary. (See the following illustration.) Units were represented by dots and fives by horizontal bars, so that the number



From the Dresden Codex of the Maya, displaying numbers. The second column on the left, reading down from above, displays the numbers 9, 9, 16, 0, 0, which stand for $9 \times 144{,}000 + 9 \times 7{,}200 + 16 \times 360 + 0 + 0 = 1{,}366{,}560$. In the third column are the numerals 9, 9, 9, 16, 0, representing 1,364,360. The original appears in black and red. (Taken from Morley 1915, p. 266.)

17, for example, would appear as  (that is, as $3(5) + 2$). A vertical positional arrangement was used, with the larger units of time above; hence, the notation  denoted 352 (that is, $17(20) + 12$). Because the system was primarily for counting days within a calendar that had 360 days in a year, the third position usually did not represent multiples of (20)(20), as in a pure vigesimal system, but (18)(20). Beyond this point, however, the base 20 again prevailed. Within this positional notation, the Maya indicated missing positions through the use of a symbol, which appeared in variant forms, somewhat resembling a half-open eye.

In their scheme, then, the notation  denoted $17(20 \cdot 18 \cdot 20) + 0(18 \cdot 20) + 13(20) + 0$.

Number Language and Counting

It is generally believed that the development of language was essential to the rise of abstract mathematical thinking. Yet words expressing numerical ideas were slow in arising. Number *signs* probably preceded number *words*, for it is easier to cut notches in a stick than it is to establish a well-modulated phrase to identify a number. Had the problem of language not been so difficult, rivals to the decimal system might have made greater headway. The base 5, for example, was one of the earliest to leave behind some tangible written evidence, but by the time that language became formalized, 10 had gained the upper hand. The modern languages of today are built almost without exception around the base 10, so that the number 13, for example, is not described as 3 and 5 and 5, but as 3 and 10. The tardiness in the development of language to cover abstractions such as number is also seen in the fact that primitive numerical verbal expressions invariably refer to specific concrete collections—such as "two fishes" or "two clubs"—and later some such phrase would be adopted conventionally to indicate all sets of two objects. The tendency for language to develop from the concrete to the abstract is seen in many of our present-day measures of length. The height of a horse is measured in "hands," and the words "foot" and "ell" (or elbow) have similarly been derived from parts of the body.

The thousands of years required for man to separate out the abstract concepts from repeated concrete situations testify to the difficulties that must have been experienced in laying even a very primitive basis for mathematics. Moreover, there are a great many unanswered questions relating to the origins of mathematics. It is usually assumed that the subject arose in answer to practical needs, but anthropological studies suggest the possibility of an alternative origin. It has been suggested that the art of counting arose in connection with primitive religious ritual and that the ordinal aspect preceded the quantitative concept. In ceremonial rites depicting creation myths, it was necessary to call the participants onto the scene in a specific order, and perhaps counting was invented to take care of this problem. If theories of the ritual origin of counting are correct, the concept of the ordinal number may have preceded that of the cardinal number. Moreover, such an origin would tend to point to the possibility that counting stemmed from a unique origin, spreading subsequently to other areas of the world. This view, although far from established, would be in harmony with the ritual division of the integers into odd and even, the former being regarded as male, the latter as female. Such distinctions were known to civilizations in all corners of the earth, and myths regarding the male and female numbers have been remarkably persistent.

The concept of the whole number is one of the oldest in mathematics, and its origin is shrouded in the mists of prehistoric antiquity. The notion of a rational fraction, however, developed relatively late and was not in general closely related to systems for the integers. Among nonliterate tribes, there seems to have been virtually no need for fractions. For quantitative needs, the practical person can choose units that are sufficiently small to obviate the necessity of using fractions. Hence, there was no orderly advance from binary to quinary to decimal fractions, and the dominance of decimal fractions is essentially the product of the modern age.

Spatial Relationships

Statements about the origins of mathematics, whether of arithmetic or geometry, are of necessity hazardous, for the beginnings of the subject are older than the art of writing. It is only during the last half-dozen millennia, in a passage that may have spanned thousands of millennia, that human beings have been able to put their records and thoughts into written form. For data about the prehistoric age, we must depend on interpretations based on the few surviving artifacts, on evidence provided by current anthropology, and on a conjectural backward extrapolation from surviving documents. Neolithic peoples may have had little leisure and little need for surveying, yet their drawings and designs suggest a concern for spatial relationships that paved the way for geometry. Pottery, weaving, and basketry show instances of congruence and symmetry, which are in essence parts of elementary geometry, and they appear on every continent. Moreover, simple sequences in design, such as that in Fig. 1.1, suggest a sort of applied group theory, as well as

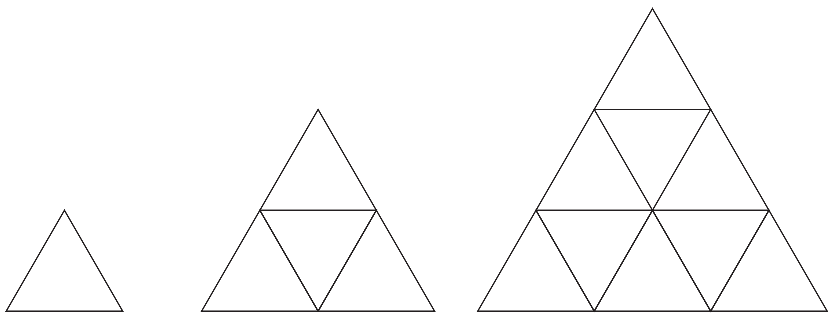


FIG. 1.1

propositions in geometry and arithmetic. The design makes it immediately obvious that the areas of triangles are to one another as squares on a side, or, through counting, that the sums of consecutive odd numbers, beginning from unity, are perfect squares. For the prehistoric period there are no documents; hence, it is impossible to trace the evolution of mathematics from a specific design to a familiar theorem. But ideas are like hardy spores, and sometimes the presumed origin of a concept may be only the reappearance of a much more ancient idea that had lain dormant.

The concern of prehistoric humans for spatial designs and relationships may have stemmed from their aesthetic feeling and the enjoyment of beauty of form, motives that often actuate the mathematician of today. We would like to think that at least some of the early geometers pursued their work for the sheer joy of doing mathematics, rather than as a practical aid in mensuration, but there are alternative theories. One of these is that geometry, like counting, had an origin in primitive ritualistic practice. Yet the theory of the origin of geometry in a secularization of ritualistic practice is by no means established. The development of geometry may just as well have been stimulated by the practical needs of construction and surveying or by an aesthetic feeling for design and order.

We can make conjectures about what led people of the Stone Age to count, to measure, and to draw. That the beginnings of mathematics are older than the oldest civilizations is clear. To go further and categorically identify a specific origin in space or time, however, is to mistake conjecture for history. It is best to suspend judgment on this matter and to move on to the safer ground of the history of mathematics as found in the written documents that have come down to us.

2

Ancient Egypt

Sesostris . . . made a division of the soil of Egypt among the inhabitants. . . . If the river carried away any portion of a man's lot, . . . the king sent persons to examine, and determine by measurement the exact extent of the loss. . . . From this practice, I think, geometry first came to be known in Egypt, whence it passed into Greece.

—Herodotus

The Era and the Sources

About 450 BCE, Herodotus, the inveterate Greek traveler and narrative historian, visited Egypt. He viewed ancient monuments, interviewed priests, and observed the majesty of the Nile and the achievements of those working along its banks. His resulting account would become a cornerstone for the narrative of Egypt's ancient history. When it came to mathematics, he held that geometry had originated in Egypt, for he believed that the subject had arisen there from the practical need for resurveying after the annual flooding of the river valley. A century later, the philosopher Aristotle speculated on the same subject and attributed the Egyptians' pursuit of geometry to the existence of a priestly leisure class. The debate, extending