

SPRINGER BRIEFS IN PHYSICS

Vladimir G. Plekhanov

Isotope-Based Quantum Information



Springer

SpringerBriefs in Physics

Editorial Board

Egor Babaev, University of Massachusetts, USA

Malcolm Bremer, University of Bristol, UK

Xavier Calmet, University of Sussex, UK

Francesca Di Lodovico, Queen Mary University of London, London, UK

Maarten Hoogerland, University of Auckland, Auckland, New Zealand

Eric Le Ru, Victoria University of Wellington, Wellington, New Zealand

James Overduin, Towson University, USA

Vesselin Petkov, Concordia University, Canada

Charles H.-T. Wang, University of Aberdeen, UK

Andrew Whitaker, Queen's University Belfast, UK

For further volumes:

<http://www.springer.com/series/8902>

Vladimir G. Plekhanov

Isotope-Based Quantum Information

 Springer

Vladimir G. Plekhanov
Mathematics and Physics Department
Computer Science College
Erika Street 7a
10416 Tallinn
Estonia

ISSN 2191-5423 ISSN 2191-5431 (electronic)
ISBN 978-3-642-28749-7 ISBN 978-3-642-28750-3 (eBook)
DOI 10.1007/978-3-642-28750-3
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2012937298

© The Author(s) 2012

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

During the past decades the field of *quantum information* processing has experienced extremely rapid progress. This book provides an introduction to the main ideas and techniques of the rapid progressing field of quantum information and quantum computation using *isotope-mixed* materials. This book is divided into four chapters. [Chapter 2](#) presents the introduction to the physics of isotope effect in solids. My goal here is to give an elementary introduction which is accessible not only to physics, but also to mathematicians and computer scientists desiring an initiation into subject. In this chapter isotope *low-dimensional structures* are very shortly described. The reader might understand the material presented in this chapter without the need for consulting other texts. [Chapter 3](#) is devoted to the description of classical and quantum information. The rest of the chapter has presented the concepts and models of quantum computers. There are discussed not only different algorithms of quantum computation but also are presented the different models of *quantum computers*. The quantum error corrections is very briefly discussed. We did not attempt to make our small book self-contained by explaining every concept which is needed only occasionally. We do hope, however, that we have succeeded in explaining the basic concepts from quantum mechanics and computer science which are used throughout the book and the whole field of quantum information and *quantum computation*.

With numerous illustrations this small book will be of great interest to undergraduate and graduate students taking courses in *mesoscopic* physics or nano-electronics as well as quantum information, and academic and industrial researchers working in this field.

The bibliography at the end of the each chapter includes many of the key papers in the area and points to other books and survey papers on the subject.

Tallinn

Vladimir G. Plekhanov

Acknowledgments

Many thanks are due to Prof. W. Reder for carefully reading the manuscript as well as Dr. N. Write for improving my English. I appreciate that invaluable help given by Mr. M. T. Kivi (Dr. of Medicine) during my difficult period. Again it is pleasure to thank the Staff of Springer, in particular Dr. C. Ascheron and Elke Sauer, for the continued excellent cooperation. I deeply thank the authors and publishers who have kindly permitted us to reproduce figures and tables from their papers and books.

In a few cases I have been unable to contact the authors, and I would be grateful if they would nevertheless retrospectively give me the necessary permission. I wish to express me deep gratitude my family for their patience during long preparation of this book.

Contents

1	Introduction	1
	References	4
2	Introduction to Isotope Effect	7
2.1	The Nucleons and its Constituents	7
2.1.1	Mass and Nuclear Binding Energy	10
2.2	Manifestation Isotope Effect in Condensed Matter	15
2.2.1	Isotope Effect in Phonon Spectra	15
2.2.2	Renormalization of Electron (Exciton) States	23
2.3	Isotope Low-Dimensional Structure	29
2.4	Excitons and Biexcitons in Quantum Dots	33
	References	39
3	Classical and Quantum Information	45
3.1	General Remarks	45
3.2	Classical Information	46
3.2.1	Shannon Entropy	47
3.2.2	Von Neumann Entropy	50
3.2.3	Introduction in Quantum Information and Quantum Computation	52
3.2.4	Information is Physical	54
3.2.5	Quantum Computation	57
3.2.6	Quantum Teleportation	58
3.2.7	Quantum Cryptography	66
3.3	Quantum Communication	69
	References	72
4	Concepts of Quantum Computers	77
4.1	Introduction	77
4.2	Current Status: The Di Vincenzo Criteria	78

4.3	Elementary Gates for Quantum Computation	79
4.4	Spintronics	87
4.5	An Introduction to Quantum Algorithms	92
4.5.1	Background	92
4.5.2	The Deutsch–Jozsa Algorithm	92
4.5.3	Simon’s Algorithm	95
4.5.4	Grover’s Algorithm	96
4.5.5	Shor’s Factorization Algorithm	97
4.6	A Physical Models for a Quantum Computer	102
4.6.1	Liquid State NMR Quantum Computer	102
4.6.2	Trapped Ions and Atoms	104
4.6.3	Solid State Quantum Computers	106
4.7	Quantum Error Corrections	114
	References	118
Index	125

Chapter 1

Introduction

Investigation, manufacture, and application of *isotopes* are highly variable and is determined by the different areas of science and technique. The range of the application of isotopes is exclusively wide: starting with the investigation of universal principle of the structure matter and common normality evolution of *Universe* [1–3] and finished by different biochemical process in living organisms as well as special technical applications [4]. The presence of isotopes and isotope effect in nature serves the bright illustration of the mutual connection between simplicity and complexity in science [5].

The paramount meaning has the role of isotopes in the fundamental natural science investigations. This includes not only the study of nature's nuclear interactions and, in this way, the origin of *isotope effect*, but also the reconstructions of nucleogenesis process of the Universe, which could explain the observable in nature relative to spreading of chemical elements [1–3].

Investigations of the *atomic nucleus*, and the fundamental forces that determine *nuclear* structure, as is well known, offer fascinating insights into the nature of the physical world. We all well known that the history of the nuclear physics dates from the latter years of the nineteenth century when Henry Becquerel in 1896 discovered the radioactivity. He was working with compounds containing the element uranium. Becquerel found that photographic plates covered to keep out light became fogged, or partially exposed, when these uranium compounds were anywhere near the plates. Two years after Becquerel's discovery, Pierre and Marie Curie in France and Rutherford in England succeeded in separating a naturally occurring radioactive element, radium ($Z = 88$) from the ore. It was soon revealed that there are three, distinctly different types of radiation emitted by radioactive substances. They were called *alpha* (α), *beta* (β) and *gamma* (γ) rays—terms which have been retained in ours days. When a *radioactive source* was placed in a magnetic field, it was found that there were three different types of activity, as the trajectories of some of the rays emitted were deflected to one direction, some to the opposite direction, and some not affected at all. Subsequently it was found that α -rays consist of positively charged ${}^4\text{He}$ nuclei, β -rays are made of electrons (positrons) and γ -rays are nothing

but electromagnetic radiation that carries no net charge. The existence of the nucleus as the small central part of an atom was first proposed by Rutherford in 1911. Rutherford proposed that the atom does consist of a small, heavy positively charged center surrounded by orbiting electrons which occupy the vast bulk of the atoms volume. The simplest atom—hydrogen—consisted of a proton and a single orbital electron. Later, in 1920, the radii of a few heavy nuclei were measured by Chadwick and were found to be in the order of 10^{-14} m., much smaller than the order of 10^{-10} m for atomic radii (for details, see e.g. [6] and references therein).

The building blocks of nuclei are *neutrons* and *protons*, two aspects, or quantum states, of the same particle, the nucleon. Since a neutron does not carry any net electric charge and is unstable as an isolated particle (see, below), it was not discovered until 1932 by Chadwick, whose existence has been anticipated by Rutherford as early as 1920. Since only positive charges (protons) are present in nucleus, the electromagnetic force inside a nucleus is repulsive and the *nucleons* cannot be held together unless there is another source of force that is attractive and stronger than Coulomb's.

Studies of the structure of the nucleus have shown that it is composed of protons and neutrons, and more recently studies (see, e.g. [6]) of very high energy collisions have shown that these protons and neutrons are themselves composed of elusive particles called *quarks*. Particle physics deals with the world of the quarks and all other particles still thought to be fundamental.

Thus, our present knowledge of physical phenomena suggests that there are four types of forces between physical objects:

- (1) gravitational;
- (2) electromagnetic;
- (3) strong, and
- (4) weak.

Both gravitational and electromagnetic forces are infinite in range and their interaction strength diminish with the square of the distance of separation. Clearly, nuclear force cannot follow the same radial dependence. Being much stronger, it would have pulled the nucleons in different nuclei together into a single unit and destroyed all the atomic structure we are familiar with. In fact, nuclear force has a very short distance.

If in the nuclear physics the meaning of isotope is establishing one then application isotope effect in atomic and molecular physics allows to get the results, which are difficult to overestimate so far as owing to this results it was to construct the “building” of the science of the twentieth century—the *quantum mechanics* (see, also [6]). During the last fifty years the isotope effect is one of the modern and power methods for investigation of structure and properties of solids. This conclusion supports the numerous reviews (see, e.g. [7–9]) and first monographs [4, 10] dedicated to isotope effect of stable isotopes. In the last years the more and more investigations of *solid-state physics* are conducted by using *radioactive* isotopes, which give evidence of the already comprehensive list of references (see, for instance [11–15]).

This book consists of four part. First one is the traditional introduction of the subject written. The second part devotes to the short description of the ground of

nature of isotope effect. With this aim the detailed analysis of the neutron and proton structure and their mutual transformation in the weak interaction process was conducted. Note that the main characteristics of isotope effect—the mass of free particles (proton and neutron) does not conserve in the *weak interaction* process. This contradiction is removed although partly if take into account the modern presentation [16] that the mass of proton (neutron) is created from quark condensate (not from constituent quarks [17–19]) which is the coherent superposition of the states with different chirality. Thus the elucidation of the reason of origin of the nucleon mass is taken down to elucidation of the reason to break down the chiral symmetry in *Quantum Chromodynamics* [20–27]. In this part of the book the manifestation of isotope effect in phonon and electron (exciton) states of solids is considered. With comparison to the change of corresponding characteristics (for example: the lines shift in *absorption, scattering, emission* spectra) in the isotope effect in atomic physics and condensed matter physics on two orders more in solid (see, for example [28]). It is underlined that taking into account only linear part of electron–phonon interaction is not sufficient for the description of the experimental facts on the elementary excitations of systems consisting of light elements with isotope effect.

The subject of *quantum information* brings together ideas from quantum physics, classical information theory, and computer science. This topic is devoted the third part of book. It is very significant that information can be expressed in different ways without losing its essential nature, since this leads to the possibility of the automatic manipulation of information—a machine needs only to be able to manipulate quite simple things like integers in order to do surprisingly powerful information processing. It is easy to do from document preparation to differential calculus and even to translating between human languages.

We should recall that quantum mechanics has developed originally as a theory to explain behavior of large number (ensembles) of microscopic objects, such as atoms or electrons [29–31]. However, over the last decades, considerable interest in the application of quantum theory to individual systems—mesoscopic and even macroscopic systems where a small number of collective degree of freedom show genuine quantum behavior (see, e.g. [32, 33]). One exciting aspect of this developing fundamental research is its technological potential. Its results that might be termed quantum information technology. As we know well, the first deep insight into quantum information theory came with Bell’s 1964 analysis [34, 35] of the paradoxical thought experiment by Einstein and co-workers in 1935 [36]. Bell’s inequality draws attention to the importance of correlations between separated quantum objects which have interacted in the past, but which no longer influence one another. In essence, his argument shows that the degree of correlation which can be present in such systems exceeds that which could be predicted on the basis of any law of physics which describes particles in terms of classical variables rather than quantum states. The next link between quantum mechanics and information theory came about when it was realized that simple properties of quantum systems, such as the unavoidable disturbance involved in measurement, could be put to practical use in quantum cryptography (see, e.g. review [38] and references therein). Quantum cryptography covers several ideas, of which the most firmly established is quantum

key distribution. This is an ingenious method in which transmitted quantum states are used to perform a very particular communication task. The significant feature is that the principles of quantum mechanics guarantee a type of conservation of quantum information, so that if the necessary quantum information arrives at the parties wishing to establish a random key. They can be sure it has not gone elsewhere, such as to spy. This part of the book considers not only the theory of *cryptology* but also its practical application [38].

References

1. E.M. Burbidge, G.R. Burbidge, W.A. Fowler, F. Hoyle, Synthesis of the elements in Stars. *Rev. Mod. Phys.* **29**, 547–652 (1957)
2. G. Wallerstein, I. Jhen Jr, P. Parker et al., Synthesis of the elements in stars: forty years in progress. *Rev. Mod. Phys.* **69**, 995–1084 (1997)
3. S. Esposito, Primordial Nucleosynthesis: Accurate Prediction for Light Element Abundances, ArXiv:astro-ph/9904411
4. V.G. Plekhanov, *Applications of the Isotopic Effect in Solids* (Springer, Heidelberg, 2004)
5. M. Gell-Mann, *The Quark and the Jaguar (Adventures in the Simple and the Complex)* (W.H. Freeman and Co., New York, 1997)
6. V.G. Plekhanov, Manifestation and origin of the isotope effect, ArXiv: gen. phys/0907.2024 (2009) p. 1–192
7. V.G. Plekhanov, Elementary excitations in isotope-mixed crystals. *Phys. Reports* **410**, 1–235 (2005)
8. M. Cardona, M.L.W. Thewalt, Isotope effect on the optical spectra of semiconductors. *Rev. Mod. Phys.* **77**, 1173–1224 (2005)
9. V.G. Plekhanov, Fundamentals and applications of isotope effect in solids. *Progr. Mat. Science* **51**, 287–426 (2006)
10. V.G. Plekhanov, *Giant Isotope Effect in Solids* (Stefan-University Press, La Jola, 2004). (USA)
11. V.G. Plekhanov, Applications of isotope effects in solids. *J. Mater. Science* **38**, 3341–3429 (2003)
12. G. Schatz, A. Weidinger, A. Gardener, *Nuclear Condensed Matter Physics*, 2nd edn. (Wiley, New York, 1996)
13. D. Forkel-Wirth, Exploring solid state physics with radioactive isotopes. *Rep. Progr. Phys.* **62**, 527–597 (1999)
14. D. Forkel-Wirth, M. Deicher, Radioactive isotopes in solid state physics, *Nucl. Phys. A* **693**, 327–341 (2001)
15. V.G. Plekhanov, *Isotope-Mixed Crystals: Fundamentals and Applications* (2011, in press)
16. F. Halzen, D. Martin, *Quarks and Leptons* (Wiley, New York, 1984)
17. E.H. Simmons, Top Physics, ArXiv, hep-ph/0011244 (2000)
18. C.D. Froggatt, The origin of mass. *Surveys High Energy Phys.* **18**, 77–99 (2003)
19. B.L. Ioffe, The origin of mass, *Usp. Fiz. Nauk (Moscow)* **176**, 1103–1104 (2006) (in Russian)
20. D.W. Lee, *Chiral Dynamics* (Gordon and Breach, New York, 1972)
21. S. Coleman, *Aspects of Symmetry* (Cambridge University Press, Cambridge, 1985)
22. G. Ecker, Chiral Perturbation Theory, ArXiv:hep-ph/9501357 (1995)
23. A. Pich, 1995, Chiral Perturbation Theory, ArXiv:hep-ph/9502366
24. S.R. Bean, P.F. Bedaque, W.C. Haxton, in *At the Frontier of Particle Handbook of QCD*, ed. by M. Shifman (World Scientific, Singapore, 2001)
25. W. Weise, Yukawa's Pion, Low-Energy QCD and Nuclear Chiral Dynamics, ArXiv: nucl-th/0704.1992 (2007)
26. J. Bijens, Chiral Perturbation Theory Beyond One Loop, ArXiv: hep-ph/0604043

27. J. Bijens, *Progr. Part. Nucl. Phys.* **58**, 521–563 (2007)
28. V.G. Plekhanov, Isotopic and disorder effects in large exciton spectroscopy, *Uspekhi-Phys.* (Moscow) **167**, 577–604 (1997) (in Russian)
29. P.A.M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, U.K., 1958)
30. R.P. Feynman, R.P. Leighton, M. Sands, *The Feynman Lecture in Physics*, vol. 3 (Addison-Wesley, Reading, MA, 1965)
31. L.D. Landau, E.M. Lifshitz, *Quantum Mechanics (Nonrelativistic Theory)* (Pergamon, New York, 1977)
32. H. Grabert, H. Horner, Eds. Special issue on single charge tunneling. *Z. Phys.* **B85**(3), 317–467 (1991)
33. T. Basche, W.E. Moerner, U.P. Wild (eds.), *Single-Molecule Optical Detection (Imaging and Spectroscopy)* (VCH, Weinheim, 1996)
34. J.S. Bell, On the EPR paradox. *Physics* **1**, 195–200 (1964)
35. J.S. Bell, On the problem of hidden variables in quantum mechanics, *Rev. Mod. Phys.* **38**, 447–452 (1966)
36. A. Einstein, B. Podolsky, N. Rosen, Can quantum mechanics description reality considered complete? *Phys. Rev.* **47**, 777–780 (1935)
37. N. Gisin, G. Ribvordy, W. Tittel, H. Zbinden, Quantum cryptography. *Rev. Mod. Phys.* **74**, 145–195 (2002)
38. G. Gibbert, M. Amrick, 2000, Practical Quantum Cryptography; A Comprehensive Analysis, ArXiv: quant-ph/ 0009027

Chapter 2

Introduction to Isotope Effect

2.1 The Nucleons and its Constituents

An atom consists of an extremely small, positively charged nucleus (see Fig. 2.1) surrounded by a cloud of negatively charged electrons. Although typically the nucleus is less than one ten-thousandth the size of the atom, the nucleus contains more than 99.9% of the mass of the atom. Atomic nucleus is the small, central part of an atom consisting of A -nucleons, Z -protons, and N -neutrons (Fig. 2.2). The atomic mass of the nucleus, A , is equal to $Z+N$. A given element can have many different isotopes, which differ from one another by the number of neutrons contained in the nuclei [1, 2]. In a neutral atom, the number of electrons orbiting the nucleus equals the number of protons in the nucleus. As usually nuclear size is measured in fermis ($1 \text{ fm} = 10^{-15} \text{ m}$, also called femtometers). The basic properties of the atomic constituents can be read in Table 2.1.

As we can see from Table 2.1, protons have a positive charge of magnitude $e = 1.6022 \times 10^{-19} \text{ C}$ (Coulombs) equal and opposite to that of the *electron*. *Neutrons* are uncharged. Thus a neutral atom (A, Z) contains Z electrons and can be written symbolically as ${}^A_Z\text{X}_N$ (see also Fig. 2.2). Here X is chemical symbol and N is neutron number and is equal $N = A - Z$. The masses of *proton* and *neutron* are almost the same, approximately 1836 and 1839 electron masses (m_e), respectively. Apart from electric charge, the proton and neutron have almost the same properties. This is why there is a common name of them: *nucleon*. Both the proton and neutron are nucleons. As we well know the proton is denoted by letter p and the neutron by n . Chemical properties of an element are determined by the charge of its atomic *nucleus*, i.e., by the number protons (electrons). It should be added, that although it is true that the neutron has zero net charge, it is nonetheless composed of electrically charged quarks (see below), in the same way that a neutral atom is nonetheless composed of protons and electrons. As such, the neutron experiences the *electromagnetic interaction*. The net charge is zero, so if we are far enough away from the neutron that it appears to occupy no volume, then the total effect of the electric force will add up to zero. The