



PALGRAVE TEXTS IN ECONOMETRICS

Modelling Trends and Cycles in Economic Time Series

Terence C. Mills

2nd Edition

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Second Edition

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Introduction

1.1 HISTORICAL PERSPECTIVE

Modelling trends and cycles in time series has a long history in empirical economics, stretching back to the latter part of the nineteenth century. Until then, few economists recognised the existence of regular cycles in economic activity, nor the presence of longer term, secular movements. Rather than cycles, they tended to think in terms of ‘crises’, used to mean either a financial panic or a period of deep depression. The early studies of business cycles, notably the sunspot and Venus theories of Jevons and Moore and the rather more conventional credit cycle theory of Jugler, are discussed in detail in Morgan (1990). The analysis of secular movements was even rarer, a notable example being Poynting (1884), who was the first to introduce moving averages. Although such movements are nowadays typically referred to as trends, the term ‘trend’ was only coined in 1901 by Hooker (1901) when analysing British import and export data. The early attempts at modelling trend movements, usually by detrending using simple moving averages or graphical interpolation, are analysed by Klein (1997).

The first quarter of the twentieth century saw great progress in business cycle research, most notably in the two ground-breaking books of Mitchell (1913, 1927) and in the periodogram studies of weather and harvest cycles by Beveridge (1920, 1921). Trends, on the other hand,

were usually only isolated so that they could be eliminated. This was still typically achieved by modelling the trend as a moving average spanning the assumed period of the cycle, or by fitting a trend line or some other simple deterministic function of time (see Mills & Patterson, 2015, for historical discussion). A notable example of this approach was Kitchin (1923), who analysed both cyclical and trend movements in data taken from the US and Great Britain over the period from 1800. Kitchin concluded that business cycles averaged 40 months in length (the Kitchin cycle) and that trade cycles were aggregates of usually two, or sometimes three, of these business cycles. Of equal interest is his conclusion that there had been several trend breaks—the final one, marking the commencement of a downward trend, occurring in 1920. A later study of secular trend, by Frickey (1934), gives a taste of the variety of methods then available for fitting trends, with twenty-three different methods used to fit a trend to pig-iron production from 1854 to 1926. Cycles were then constructed by residual, producing average cycles ranging in length from 3.3 to 45 years, thus showing how the observed properties of cyclical fluctuations could be totally dependent on the type of function used to detrend the observed data.

This research was primarily descriptive and statistical. The late 1930s, in contrast, saw the development of formal models of cyclical fluctuations in the economy. Three classic examples were Samuelson's (1939) analysis of the interactions between the multiplier and the accelerator using solutions to difference equations, Kaldor's (1940) primarily diagrammatic, but nonlinear, model of the trade cycle, and Metzler (1941) who, using techniques similar to Samuelson, investigated the role of cyclical fluctuations to inventories in producing business cycles.

While these theoretical developments were taking place, various critiques of business cycle research were being formulated. These took several forms, beginning with the use of statistical analysis to attack the very foundations of business cycles. Fisher (1925) investigated whether fluctuations in the price of the dollar were a primary cause of trade fluctuations. He was particularly innovative in the statistical techniques that he used, as he was the first to introduce distributed lag structures into regression analysis. Fisher argued that, because of the high correlation between price changes and subsequent movements in trade volumes and the lack of cycles in the residuals from this relationship, the business cycle as a normal set of ups and downs in the economy did not exist. Unfortunately, the sample used by Fisher, 1915–1923, was probably too short

to warrant such conclusions, and subsequent reworking of the data by Hendry and Morgan (1995, pages 45–48) suggests that his statistical techniques, although undeniably innovative, were somewhat flawed and did not support the conclusions that he claimed to have reached.

Further difficulties with analysing economic data that appeared to exhibit cyclical behaviour were emphasised by the research of Yule (1926) and Slutsky ([1927] 1937). Yule showed that uncritical use of correlation and harmonic analysis, both very popular at the time, was rather dangerous, as ignoring serial correlation in, and random disturbances to, time series could easily lead to erroneous claims of significance and evidence of harmonic motion. Slutsky investigated a more fundamental problem—that observed cycles in a time series could be caused entirely by the cumulation of random events. Slutsky’s research was not primarily aimed at analysing business cycles, but Kuznets (1929) took up this issue, using simulation and graphical techniques to explore which shapes of distributions of random causes, which periods of moving averages, and which weighting systems produced the most cyclical effects. Indeed, Kuznets pointed out that this analysis not only removed the necessity for having a periodic cause for economic cycles but could also make further discussion of the causes of business cycles superfluous.

These studies paved the way for the development of the first detailed dynamic models of the business cycle. Frisch’s (1933) influential ‘rocking horse theory’ of the business cycle was built on the ideas of Yule and Slutsky (see also Frisch, 1939). Frisch was also keen to suggest that the parameters of business cycles models should be estimated using real data, rather than being chosen by guesswork, and this suggestion was taken up by Tinbergen, who built and estimated the first macrodynamic models of the business cycle, using techniques expounded in detail in Tinbergen (1939a). A model of the Dutch economy appeared first (Tinbergen, 1937), to be joined later by models of the US (Tinbergen, 1939b) and the UK (Tinbergen, 1951). While Tinbergen’s Dutch model made little impact, his first report for the League of Nations (Tinbergen, 1939a) provoked a long-lasting discussion on the role of econometrics in the testing of economic theory. This debate was sparked off by Maynard Keynes’ famous review in the *Economic Journal* (Keynes, 1939). To those economists who had not read Tinbergen’s report and who remained ignorant of developments in econometrics since the mid-1920s, Keynes’ review must have represented a devastating criticism. After the publication of Tinbergen’s (1940) response, and subsequent contributions by

Tinbergen (1942) and Haavelmo (1943), a rather different view began to take hold. As Hendry and Morgan (1995, page 54) later remark, ‘the first suspicion is that Keynes might have been reading another book altogether, or at least did not read all of the book’, something that Tinbergen suggested in his reply! While there were many difficulties in empirically implementing econometric models of the business cycle, Tinbergen’s research was a tremendous step forward and laid the foundation for much of the macroeconomic modelling that subsequently took place.

In the aftermath of World War II, 1946 saw the publication of Burns and Mitchell’s magnum opus for the National Bureau of Economic Research (NBER), in which they produced a new set of statistical measures of the business cycle, known as specific cycles and reference cycles, and used these to test a number of hypotheses about the long-term behaviour of economic cycles (Burns & Mitchell, 1946). This volume created a great deal of interest and provoked the review of Koopmans (1947), which initiated the famous ‘measurement without theory’ debate in which he accused Burns and Mitchell of trying to measure economic cycles without having any economic theory about how the cycle worked. Koopmans’ review in turn provoked Vining’s (1949) defence of the Burns and Mitchell position, in which he charged Koopmans with arguing from a rather narrow methodological position, that associated with the ‘Cowles group’, which had yet to demonstrate any success in actual empirical research.

Although the ‘measurement without theory’ debate obviously focused on the measurement and theoretical modelling of business cycles, some disquiet had also been expressed, particularly by Ames (1948), about the role of secular trends and the methods by which they were removed before cyclical fluctuations could come to the forefront of the analysis. The appropriate way of modelling trends was later to become a prominent theme in macroeconomic research, but the early 1950s saw theorists begin to work on models in which trend and cycle could interact, two particularly influential examples being Kaldor (1954) and Higgins (1955).

The stage was now set for the modern development of theories of the business cycle. Progress was, however, somewhat inhibited during the 1950s and 1960s, for the sustained growth of the leading world economies during this period (the ‘golden age’ of economic growth) drew attention away from analyses of cyclical fluctuations and towards those of demand management and fine tuning of the economy. Even a

new definition of cyclical fluctuations was proposed—‘growth cycles’—which were the deviations from long trends rather than levels of economic aggregates. The distinction between growth cycles and business cycles is carefully pointed out in the major survey by Zarnowitz (1985).

The last quarter of the twentieth century, however, saw the development of several new classes of business cycle models. During the 1970s, after the end of the golden age ushered in a period of great economic and political instability across many western economies, the concept of a political business cycle became popular. The original model of this type was that by Nordhaus (1975). The basis of these models is the notion that governments adopt monetary and fiscal policies so as to maximise their chances for re-election, given that the typical cycle is roughly the same duration as the term of office of the policymakers. Prior to an election, the government will do all it can to stimulate the economy. The negative consequences of these policies will not be felt, of course, until more than a year after the election, when they must then be reversed. This suggests that an electoral-economic cycle will be discerned in, for example, output and unemployment.

Published contemporaneously with Nordhaus’ political business cycle model was the radically different approach of Lucas (1975). In this famous application of the rational expectations’ hypothesis to macroeconomics, Lucas developed a general business cycle theory that adheres strictly to the basic principles of the analysis of economic equilibrium, i.e., the consistent pursuit of self-interest by individuals and the continuous clearing of all markets by relative prices. This paper led to a large literature on rational expectations models of the business cycle, which in turn prompted the development of real business cycle (RBC) models. This literature is exemplified by Kydland and Prescott’s (1982) prototype RBC model, where a single technology shock to the production function is the source of the stochastic behaviour of all the endogenous variables in the model. This model represents an integration of neoclassical growth theory (as exemplified by Solow, 1970) with business cycle theory by replacing the constant returns to scale neoclassical production function with stochastic elements and a ‘time to build’ technology, so that multiple periods are required to build new capital goods and only finished capital goods are part of the productive capital stock. Long and Plosser (1983) provided a model that is richer than the Kydland and Prescott

prototype in that it adopts a sectoral approach to production, with input–output relationships propagating the effects of stochastic output shocks both forward in time and across sectors.

A further class of business cycle models is based on the twin ideas of comovement of contemporaneous economic time series via common shocks (known as factor structure), and regime switching between ‘good’ and ‘bad’ states: see, for example, the dynamic factor model of Stock and Watson (1991), the Markov regime-switching set-up of Hamilton (1989), and the synthesis of the two approaches by Diebold and Rudebusch (1996).

Other research areas also prospered, often in response to popular developments in allied disciplines. For example, the 1970s also saw the development of catastrophe theory, first to describe biological processes and then applied to other areas (see, for example, Zeeman, 1977). Varian (1979) used catastrophe theory to examine a variant of Kaldor’s trade cycle model, showing that a small shock to one of the stock variables will produce a minor recession in inventories, but that a large shock may lead to such a decline in wealth that the propensity to save is affected and the subsequent very slow recovery can result in a deep depression.

Nonlinearities can occur in business cycles in many ways. One important form of nonlinearity is that of asymmetry. An asymmetric cycle is one in which some phase of the cycle is different from the mirror image of the opposite phase: for example, contractions might be steeper, on average, than expansions. Although such asymmetries were noted by early business cycle researchers (for example, Kaldor’s 1940 model yields asymmetric cycles, while Burns & Mitchell, 1946, actually observed them in US data), methods for formally examining asymmetries were only developed much later. Neftçi (1984) was the initial attempt, uncovering evidence of asymmetry in US unemployment by using a nonparametric procedure. Stock (1987) extended these ideas to consider whether macroeconomic variables do indeed evolve on a cyclical time scale, i.e., as defined by turning points rather than by months and quarters (the calendar time scale), or whether they evolve on a different scale altogether. The former view is implicit in the analysis of Burns and Mitchell, but Stock found evidence that, although US macroeconomic data evolves on an ‘economic’ rather than a calendar time scale, the estimated economic time scales are only weakly related to those of the business cycle.

As catastrophe theory became popular in the 1970s, so did chaos theory a decade later. Chaotic dynamics are nonlinear deterministic movements through time that appear random when subjected to standard statistical tests (see, for example, Baumol & Benhabib, 1989). Brock and Sayers (1988) applied the new tests for chaotic dynamics to macroeconomic data and, although they found much evidence of nonlinearity, no conclusive evidence of chaos was obtained, a situation that still pertains today. The duration dependence of business cycles has also been investigated. Duration dependence is the idea that expansions and contractions die of old age, i.e., that business cycle regimes are more likely to end as they get longer, so that business cycle lengths tend to cluster around a certain duration, a notion of periodicity that was long implicit in the traditional business cycle literature (see, for example, Diebold & Rudebusch, 1990).

Until fairly recently, business cycle research still tended to mention the presence of long-term trends almost in passing. As mentioned earlier, a not too distorted caricature is that data needs only to be detrended by a simple and readily available method so that attention can quickly focus on the much more interesting aspects of cyclical fluctuations. Although there are some notable exceptions, this approach is only justifiable if there is indeed little interaction between the trend growth of an economy and its short-run fluctuations. Even then, instability in the trend component and/or the use of an incorrect procedure for detrending will complicate the separation of trend from cycle. With the development of growth theory, some attention began to focus on the modelling of trends, with Klein and Kosobud (1961) representing an innovative attempt at fitting trends not just to individual series, but to certain of their ratios—the ‘great ratios’ of growth theory. This paper is arguably a forerunner of the idea of common trends that underlies the concept of cointegration, which plays such a pivotal role in modern time series econometrics, as exemplified by, for example, Banerjee et al. (1993). Mills (2009) provides a modern perspective on Klein and Kosobud’s great ratios.

Klein and Kosobud restricted their analysis to linear, or log-linear, trends, and this was a common assumption for much of the 1960s and 1970s. Although often a useful approximation, the assumption of a constant deterministic trend becomes increasingly implausible over long historical periods, where there are likely to be structural changes in the economy as well as varying rates of factor accumulation and technical progress. It is therefore reasonable to entertain the notion of shifts or

breaks in trend or even period-by-period random or stochastic trends. Such trend variability complicates its separation from the cycle and incorrectly assuming a linear trend in these circumstances can lead to spurious cycles being introduced (see, for example, Nelson & Kang, 1981). These twin issues—the independence and the variability of trends—has been the subject of great debate over the past four decades, a debate that was initiated primarily by two papers, Beveridge and Nelson (1981) and Nelson and Plosser (1982). The former focuses on how to separate trend and cycle when the series is generated by an integrated, or difference stationary, process, i.e., one that has no tendency to return to a deterministic linear trend but evolves as a drifting, and possibly correlated, random walk. The latter paper utilises techniques developed by Dickey and Fuller (1979) to test whether time series are indeed difference stationary rather than trend stationary (i.e., ones that do indeed tend to return to a deterministic linear trend). They applied these tests to a set of US macroeconomic time series and found that the evidence was heavily in favour of the difference stationary representation.

Although many researchers embraced the stochastic trends view of macroeconomic dynamics embodied in these papers, not all economists and econometricians were persuaded by a universal finding of difference stationarity in macroeconomic time series (or the presence of unit roots, as it is also referred to). Alternative testing techniques, usually either small sample methods (see, for example, Rudebusch, 1992) or those based on a Bayesian perspective (DeJong & Whiteman, 1991), tended to offer evidence more in favour of trend stationary formulations in the Nelson and Plosser data set. Alternative trend formulations have also been considered. One particularly interesting approach in the context of modelling trends and cycles is the possibility that a unit root appears as a consequence of failing to model the underlying trend as a nonlinear, rather than a linear, function of time. A realistic model may be one in which a linear trend is subject to occasional shifts, possibly in both level and slope, that are produced by infrequent permanent shocks that either occur exogenously (Perron, 1989) or can arrive randomly (Balke & Fomby, 1991).

It is now well known that cointegration between a set of difference stationary series results in them being driven by a reduced number of common stochastic trends. For already stationary series, an analogous property would be that a linear combination of autocorrelated variables has less autocorrelation than any of the individual series (in the sense

of having an autocorrelation function that decays to zero quicker). For example, a linear combination of stationary but autocorrelated series could itself be white noise, in which case we say that the individual series share a common cycle (more generally, a common feature, in the terminology of Engle & Kozicki, 1993). Vahid and Engle (1993) showed how the common trend formulation can be extended to incorporate common cycles, as well as providing a framework in which both types of restrictions can be tested and imposed sequentially, thus allowing an integrated analysis of trends and cycles to be undertaken.

This recent emphasis on modelling trends has led to a renewed interest in issues of detrending. A formal treatment of the issue casts the trend extraction problem in an unobserved component framework similar to that of Harvey (1985) and uses signal extraction techniques to estimate the trend and cycle. Although this will produce optimal detrending if the forms of the unobserved components are known, this may be too stringent a requirement in many applied situations. There have thus been various attempts to construct trend estimators that work well in a variety of situations. Perhaps the most popular of these is that proposed by Hodrick and Prescott (1997). Originally circulated as a working paper in 1980, it was eventually published as a journal article some seventeen years later, although by then it had been used in hundreds of applications! This estimator is known as the Hodrick-Prescott (H-P) filter, because it is a two-sided weighted moving average (or filter) whose weights are obtained from a particular optimisation problem—that of minimising the variance of the cyclical component subject to a penalty for variation in the second difference of the trend component; in other words, a smoothness requirement. The extent of the penalty depends on the value set for the smoothing parameter which appears in each of the weights and which is typically set at 1600 for quarterly data. The H-P filter became very popular for detrending data for use in RBC models. A critical aspect of the filter, however, is the nature and properties of the cyclical component that it produces. For example, Cogley and Nason (1995) analyse this aspect of the H-P filter and show that it can generate spurious cycles in difference stationary processes, so that the cycles observed in detrended data may simply reflect the properties of the filter and may tell us very little about the properties of the underlying data. Harvey and Jaeger (1993) make much the same point and Osborn (1995) shows that similar conclusions result from simple moving average detrending. These criticisms have recently been revisited and extended by Hamilton (2018).

It is important to emphasise that arguments about how to detrend are not equivalent to arguments about what the business cycle frequencies are. In fact, the H-P filter with smoothing parameter set at 1600 closely approximates a high-pass filter with a cut-off point of 32 cycles per period, i.e., a filter that passes frequencies up to the cut-off point, which corresponds to the usually accepted maximum length (in quarters) of a business cycle. Baxter and King (1999) develop the theory of band-pass filters (filters that pass frequencies between lower and upper bounds, usually taken to be between 6 and 32 quarters for business cycles) and propose an alternative to the H-P filter that seems to have somewhat better general properties. Nevertheless, there continues to be considerable argument about the use of filters, and indeed other detrending methods, as the debate between Canova (1998) and Burnside (1998) demonstrates.

Many of the key papers referred to in the above discussion are collected in Mills (2002), while Mills (2011, 2013) provides detailed discussion of the historical context and development. Mills (2019) provides an introductory econometric treatment of many trending mechanisms.

1.2 OVERVIEW OF THE BOOK

Chapter 2 considers ‘classical’ techniques of modelling trends, such as deterministic functions of time, including nonlinear, segmented, and smooth transition formulations, and moving averages. Autoregressive processes are introduced for modelling a cycle, and some problems associated with these techniques, such as the Slutsky-Yule effect, are discussed.

Stochastic trends are the focus of Chapter 3, where the properties and implications of integrated processes are investigated, along with the distinction between trend and difference stationarity. The class of unobserved component models is introduced, and this leads naturally to a discussion of the Beveridge-Nelson decomposition, basic structural models, and the estimation technique of signal extraction.

Chapter 4 is concerned with detrending using linear filters. Their analysis requires some familiarity with frequency-domain concepts, and the required techniques are provided in this chapter. Filter design is then considered before the popular H-P, band-pass and Butterworth filters are introduced and linked to unobserved component models.

In recent years there has been an upsurge in interest in nonlinear and nonparametric modelling in economics. Several of these techniques have

been applied to the analysis of trends in time series. Chapter 5 analyses various of the regime shift models that have been proposed for dealing with shifting trends and also considers nonparametric smoothing procedures for extracting trend components.

Up to this point the book has been concerned with procedures that operate on a single time series. Chapter 6 extends these techniques to a multivariate environment, beginning with the concept of common features, before extending the analysis to consider common trends and cycles within a vector autoregressive framework and to the concept of co-breaking. Multivariate extensions of linear filtering are then considered.

Chapter 7 presents brief conclusions and suggestions for an appropriate research strategy for modelling trends and cycles.

The techniques are illustrated by a variety of empirical examples and, rather than cluttering the exposition, citations and references to further reading are provided at the end of each chapter. All the examples use *Econometric Views (EViews)*, Version 10 and a collection of *EViews* work files containing all the data used in the examples, along with the commands required to perform computations, are included at the end of the book. It is assumed as a prerequisite that readers have a basic knowledge of statistical inference and of time series econometrics, say at the levels of Mills (2014, 2015), and an understanding of the elementary elements of matrix algebra.

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