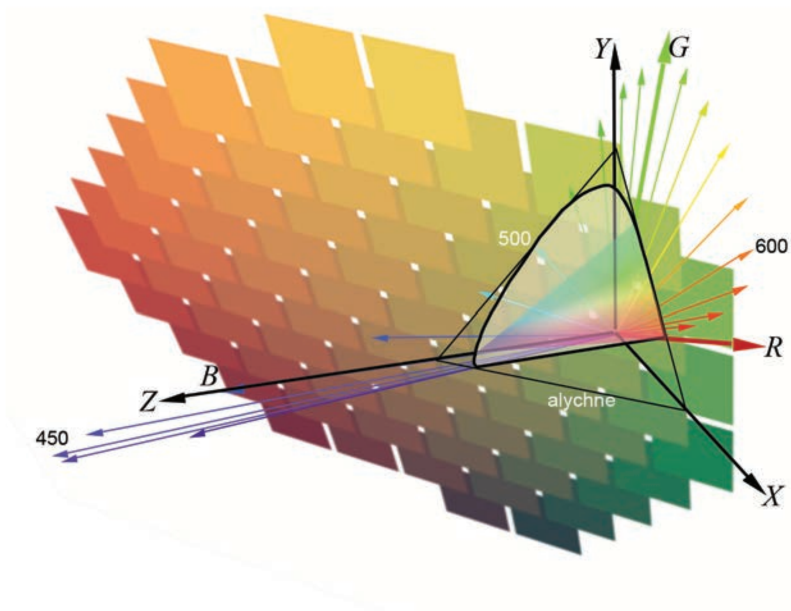
society of dyers and colourists

Standard Colorimetry

Definitions, Algorithms and Software



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Standard Colorimetry

Definitions, Algorithms and Software

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To those who strive to be just

Contents

Society of Dyers and Colourists

Society of Dyers and Colourists (SDC) is the world's leading independent, educational charity dedicated to advancing the science and technology of colour. Our mission is to educate the changing world in the science of colour.

SDC was established in 1884 and became a registered educational charity in 1962. SDC was granted a Royal Charter in 1963 and is the only organization in the world that can award the Chartered Colourist status, which remains the pinnacle of achievement for coloration professionals.

We are a global organization. With our Head Office and trading company based in Bradford, UK, we have members worldwide and regions in the UK, China, Hong Kong, India and Pakistan.

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For further information please email: info@sdc.org.uk, or visit www.sdc.org.uk.

Preface

"Standard Colorimetry" is an ambitious title that comes from the project of a small book, already fully written and never published, entitled *Concise Handbook of Standard Colorimetry*. The reviewers, who certainly knew my scientific production, suggested to broaden the content of the book, pointing me to chapters and contents. The book has become bigger, but more personal. This produced the change of the title, which contracted as *Standard Colorimetry*.

The books published in recent years on colorimetry are all excellent, comprehensive and authoritative, and written by authors and experts, and surely many readers have not felt the need for the publication of a further book. However, the differences between these books, including this one, are obvious.

Each book highlights the author's knowledge, expertise and experience, which are made of reliefs, accents that make the various points otherwise important and in this sense reveal the views of the author. These important features differentiate the various books.

I do not like to take possession of the sentences of others, so the text is full of quotations in inverted commas, indicating clearly the source. This is a way to go to the source and respect the authors.

A software that accompanies this book has the function of giving visual concreteness to the numbers that specify the colour and is a tool for all colorimetric calculations.

Today this book is the book I wish I had read in a sequential way, starting from the first row, when, at the age of about 45 years, the case led me to passionately study human colour vision.

I thank the unknown reviewers. I appreciate the quality of their work and their competence.

I thank the many colleagues that through dialogue, often with very short conversations, e-mail exchanges, or simply the seminars I attended, helped me to understand and know, led me to get a varied overview of colour science. I cannot cite everyone. I feel obliged to mention one name among them all, Robert M. Boynton, because in 2003 in a very short workshop in La Jolla he made us understand that every formula is obtained by engineering, but its value lies in its capacity to explain the phenomena and not simply to fit the phenomena. He had a high conception of science. Today there are too many formulae in colorimetry that have only a practical value but are unsatisfactory and do not help us to understand the phenomena.

Thanks to the readers who want to tell me the darkness and the errors encountered in reading the book or just want to comment. Send me suggestions and questions through e-mail: claudio.oleari@fis.unipr.it.

Claudio Oleari
2015

1

Generalities on Colour and Colorimetry

The *Commission Internationale de l'Éclairage* (CIE) is the official institution devoted to worldwide cooperation and the exchange of information on all matters relating to the science and art of light and lighting, colour and vision, photobiology and image technology.

CIE publications are the main reference for this book.[1–3] This book is about colorimetry and has the definitions of colour and colorimetry as its starting point.

1.1 Colour

In non-specialist language, the word 'colour' is ambiguous, because it is used to describe the quality of the objects, self-luminous and non-luminous, and to describe a quality of the viewing experience. These meanings of the same word 'colour' are different but they are not disjoint, because the first one is the stimulation of the visual experience and the other the visual experience itself. Between these two meanings there is a correspondence and colorimetry quantitatively describes this correspondence.

The colour of self-luminous and non-luminous objects is associated with a physical quantity, which is properly called *colour stimulus* and is measurable because it is external to the body of the observer:

> "*Colour stimulus* – visible radiation entering the eye and producing a sensation of colour, either chromatic or achromatic."[1]

The definition of colour as an effect of the colour stimulus is given by the *Optical Society of America* (OSA) in the 1952 report:

> "Color consists of the characteristics of light other than spatial and temporal inhomogeneities; light being the aspect of radiant energy of which a human being is aware through the visual sensations which arise from the stimulation of the retina of the eye."[4]

Standard Colorimetry: Definitions, Algorithms and Software, First Edition. Claudio Oleari.

Among the many definitions of colour, the most comprehensive, albeit in its brevity, is given by the *American Society for Testing and Materials* (ASTM),[5] which with the definitions opens highly technical discussions, which are clarified later in the book:

1. "*Colour of an object* – aspect of object appearance distinct from form, shape, size, position, or gloss that depends upon the spectral composition of the incident light, the spectral reflectance or transmittance of the object, and the spectral response of the observer, as well as the illuminating and viewing geometry."[5]

2. "*Perceived colour* – attribute of visual perception that can be described by colour names such as white, grey, black, yellow, brown, vivid red, deep reddish purple, or by combinations of such names. *Discussion* – perceived colour depends greatly on the spectral power distribution of the colour stimulus, but also on the size, shape, structure, and surround of the stimulus area, the state of adaptation of the observer's visual system, and the observer's experience with similar observations."[5]

The 'perceived colour' is defined using the names of the colours. This means that the names of the colours represent fundamental concepts, which are not definable in other words. The perceived colour is incommunicable. Humans evoke the perceived colour in the interlocutors with conventional words – red, yellow, green, blue, black, grey, white, so on –.

1.2 Colorimetry

Robert W. Hunt[6,7] distinguishes between:

> "*Psychophysical colour terms* **–** terms denoting objective measures of physical variables that are evaluated so as to relate to the magnitudes of important attributes of light and colour. These measures identify stimuli that produce equal responses in a visual process in specified viewing conditions."

and

> "*Psychometric colour terms* **–** terms denoting objective measures of physical variables that are evaluated so as to relate to differences between magnitudes of important attributes of light and colour and such that equal scale intervals represent approximately equal perceived differences in the attribute considered. These measures identify pairs of stimuli that produce equally perceptible differences in response in a visual process in specified viewing conditions."[6]

Psychophysical colour terms regard *Psychophysical colorimetry* and psychometric colour terms regard *Psychometric colorimetry*. Both definitions of *psychophysical* and *psychometric colour* refer to colour stimuli, whose measurement and processing are same as those in the human visual system. The human visual system is a tool that measures the colour stimulus, as a camera, (psychophysics) and processes the signals produced quantifying the colour attributes according to a perceptive scale (psychometrics).

Psychophysical colorimetry is limited to the measurement of colour stimuli, attributing the same specification to different colour stimuli which induce equal colour sensations. This is exactly what happens in a photographic camera.

The human eye, unlike the camera, has a sensor – the retina – that has not the same optical properties in all its parts. The central part, for acute vision, is different from the surrounding parts, for which, according to a simplified diagram, there are two different colorimetries. In 1931 the CIE defined a colorimetry for

Table 1.1 Scheme of the colour specification according to historical steps, stages of vision, visual fields and referred to the CIE standard systems.

		Historical steps/stages of vision/systems		
		First stage of vision: transduction	Second stage of vision: colour difference and illuminant discounting	Third stage of vision: colour appearance and adaptation
		Psychophysics (Chapter 9)	Psychometrics (Chapter 11)	Colour appearance
visual	Visual field < 4°	CIE 1931 standard observer (X, Y, Z),	CIELUV system (L^*, u^*, v^*) CIELAB system (L^*, a^*, b^*)	CIECAM97 CIECAM02 Retinex
		Vos observer 2° CIE fundamental observer		
field	10° visual field	CIE 1964 supplementary standard observer (X_{10}, Y_{10}, Z_{10})	CIELUV system $(L^*_{10}, u^*_{10}, v^*_{10})$ CIELAB system $(L^*_{10}, a^*_{10}, b^*_{10})$	
		10° CIE fundamental observer		

acute vision – observer with a visual field of 2° described in Section 9.2 – and in 1964 a colorimetry for non-acute vision – observer with the field of view of 10° described in Section 9.3 –.

The distinctions between psychophysical and psychometric colorimetries, and between the 2° and 10° visual fields, have led to four different colorimetries, as summarized in Table 1.1.

Over time, the study of colour-vision has led to improving the standard observers by adding new cases within the schema of Table 1.1, that is, Vos and fundamental observers described in Sections 9.5 and 9.6. These improvements are considered so small that the industries and laboratories continue to use the standard CIE 1931 and CIE 1964.

This distinction among different colorimetries corresponds to a distinction among the historical phases of colorimetry[7]:

A first phase concerned with 'which colours match' and is termed *psychophysical* in strict sense (Chapter 9) and also *classical colorimetry* or *tristimulus colorimetry*.

A second phase concerned with 'whether colour differences are equal' and is termed *psychometric* (Chapter 11).

A third phase concerned with 'what colours look like' and is termed "*colour appearance*:

i. aspect of visual perception by which things are recognized and
ii. in psychophysical studies, visual perception in which the spectral and geometric aspects of a visual stimulus are integrated with its illuminating and viewing environment."[1]

The third phase of colour-appearance colorimetry is in rapid progress and is a subject of debate; therefore, it is not considered in this book.

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2

Optics for Colour Stimulus

2.1 Introduction

This chapter has solely educational purposes and only recalls classical optical phenomena, therefore its bibliography consist of textbooks, manuals and tutorials.

This chapter considers the physical nature of colour stimuli.

> "*Colour stimulus* – visible radiation entering the eye and producing a sensation of colour, either chromatic or achromatic."[1–3]

> "*Colour stimulus function* ϕ_λ – description of a colour stimulus by the spectral concentration of a radiometric quantity, such as radiance or radiant power, as a function of wavelength."[1–3]

> "*Relative colour stimulus function* $\phi(\lambda)$ – relative spectral power distribution of the colour stimulus function."[1–3]

> "The psychophysical specification of a colour stimulus is termed again *colour stimulus* and is denoted by a symbol in square brackets, for example, $[\phi]$"[1–3] (the context avoids ambiguity). Since the psycophysical colour stimulus is mathematically a vector, in this book the colour stimulus is always indicated in bold roman letters as scientific convention requires.

The colour stimulus function is a physical quantity measured in W/nm if it is the spectral distribution of the radiant power – also *spectral flux* –, or in W/(m^2 sterad nm) if it is the spectral distribution of the radiance. The relative colour stimulus function is dimensionless and the spectral plots of this function and that of the colour stimulus function are proportional, that is, $\phi_\lambda = k\ \phi(\lambda)$, where k is a constant with a suitable physical dimension. The colour stimulus functions must be used for the computations of absolute

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quantities as, for example, the illuminance and the luminance, but generally the use of the relative colour stimulus function is more convenient and almost all the colour specification is made on a relative scale. [In colorimetry and photometry the absolute spectral distribution functions have dependence on the wavelength λ written as a subscript while the relative spectral distribution functions have dependence written in brackets (λ).]

These definitions of colour stimulus are merely radiometric (Section 2.4) and only physical operations on these quantities are considered, for example, addition of many colour stimuli.

Colour stimuli produce colour sensations, that usually continue to be called colour stimuli, which should not be confused with the previous colour stimuli (the context avoids ambiguity). Psychophysical operations can be made on these last colour stimuli, for example, addition, comparison and colour matching, and in this case colour stimuli, which represent colour sensations, are written in square brackets $[\psi]$ or as vectors.

The colour stimulus function characterized by a spectral power distribution constant is termed *equal-energy (*or *equi-energy) stimulus function* and denoted by the relative spectral radiance $E_E(\lambda) = 1$. The equal energy stimulus function often plays a role in the normalization of the tristimulus space, used for the psychophysical specification of colour stimuli (Section 6.13).

Colour vision is a phenomenon triggered by colour stimulus, and it happens by the interaction between visible electromagnetic radiation (light) and the visual system. Activation consists of the absorption of radiation by typical molecules located inside photosensitive cells of the retina on the fundus of the eye. Everything that happens before the activation itself concerns the interaction between the electromagnetic radiation and matter. It is therefore necessary to have a theoretical framework that describes the generation, propagation and interaction of light with matter in order:

1. to have a basis for the understanding of the meaning of colour stimulus function and of colour perception and
2. to quantify and forecast the light phenomena, and connect them with the physical nature of matter.

Point (1) is considered in this introductory section. The physical phenomena related to the colours, often called *colour physics*, are described in Chapter 3, while the fundamental optical phenomena used in colour science are concisely recalled where necessary.

It is known that visible light is an electromagnetic radiation constituted by mutually orthogonal electric and magnetic fields that propagate in space and time, in a vacuum and in matter, with a velocity dependent on the nature of the medium.

Electromagnetic radiation shows a dual nature, *corpuscular* and *wave nature*: the processes in which light is generated or absorbed can be explained assuming that it consists of quanta, postulated by Einstein and called *photons*, while light propagation in space and time is well explained by a wave-like behaviour, fully described by Maxwell's equations.

Everyday experience suggests we consider light as consisting of rays. *Geometrical optics* is based on this assumption and is derivable from wave optics by an approximation, termed *eikonal approximation*. Geometrical optics is valid only when light interacts with objects, which are much larger than its wavelength.

All these descriptions of light radiation enter the representation of vision phenomena and the lab equipment:

- The image formation in the eye is well described by geometrical optics.
- The transduction of light radiation into an electrical phenomenon in the photosensitive cells of the eye is a photochemical process and regards the quantum nature of light, that is, photons.
- The photo-detection of solid state devices, generally used in radiometric, photometric and colorimetric instruments, is often a quantum phenomenon and regards the quantum nature of light.

- The emission of light radiation from a body – light source – is a quantum process, albeit divided into two different processes:
  1. transitions between electronic states of matter at different energy – responsible for the coloration of many bodies and
  2. blackbody emission, a phenomenon dependent only on the temperature of the body (Section 3.3).
- The interaction of the electromagnetic radiation with matter is describable with one of the three descriptions (geometrical, wave and quantum optics) according to the phenomena involved.

Here we describe briefly the characteristics of light according to the wave and quantum model. (It is assumed that the reader is familiar with geometrical optics.)

2.2 Electromagnetic Waves

The wave hypothesis dates back to Huygens (1629–1695), but then there are the Maxwell equations (1873) that fully describe the light as electromagnetic waves. The electromagnetic properties of the matter, in which there are waves, are represented by the *dielectric constant* ε and *magnetic permeability* μ. In fact, these are precisely the electromagnetic properties of matter that determine the light velocity $v = (\varepsilon\mu)^{1/2}$ and consequently the optical properties. The light velocity in the vacuum, denoted by c, is the maximum possible velocity and is an universal constant of physics. The *refractive index* $n \equiv c/v$, known in geometric optics, is a quantity that summarizes these properties. The refractive index is a function of the wavelength and is typical for any material.

The waves, solutions of Maxwell's equations in an optically homogeneous medium, are represented by sinusoidal functions called *wave functions*, whose argument, the *phase* of the wave, is defined at the point x of the space along the direction of propagation and the time instant t by

$$2\pi\left(\frac{x}{\lambda}-\frac{t}{\tau}\right)-\vartheta_0=\kappa x-\omega t-\vartheta_0 \tag{2.1}$$

τ is the period;
$\nu = 1/\tau$ is the frequency;
$\omega = 2\pi\nu$ is the pulsation;
λ is the wavelength;
$\kappa = 2\pi/\lambda$ is the wave number (sometimes the wave number is simply the reciprocal of λ); and
ϑ_0 is the initial phase.

The electromagnetic waves have the following properties:

- The quantities described by wave functions are the electric field vector $\vec{E}(\kappa x-\omega t-\vartheta_0)$ and the magnetic field vector $\vec{H}(\kappa x-\omega t-\vartheta_0)$. A qualitative representation of a wave in space and time is given in Figure 2.1.
- $\vec{E}$ and $\vec{H}$ are perpendicular to each other and to the direction of propagation – *transverse waves* – (Figure 2.1).
- The waves associated with the electric filed are *plane waves* if the vectors $\vec{E}$ at any points of the propagation line belong to a plane. Since electric and magnetic fields are linked together by Maxwell's equations, the vectors $\vec{H}$ of the magnetic field belong to a plane orthogonal to the plane of the electric field. The surfaces on which the phase of the wave is constant are called *wave fronts*, and the planes tangent to these surfaces are perpendicular to the direction of propagation.

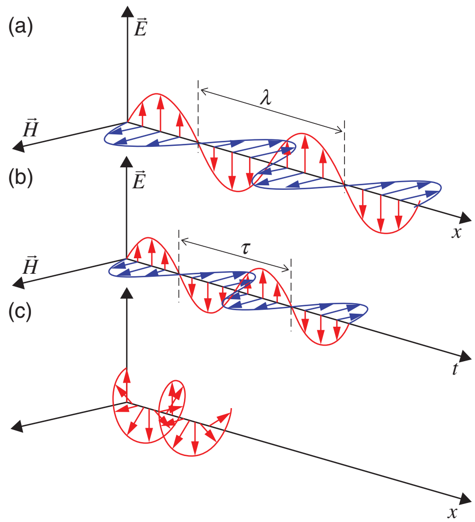

Figure 2.1 (a) Plane electromagnetic waves represented in space at fixed time and (b) in the time in a fixed point in space. This graphical representation shows the wavelength and the frequency in space and time, respectively. (c) The third graph from the top represents a wave with 'circular polarization'; here only the electric field is represented and the magnetic field is orthogonal to the electric field. The electric field and magnetic field are mutually perpendicular and together orthogonal to the propagation direction.

- A light wave is *linearly polarized* if the wave is a plane wave (Figure 2.1). Linear polarization and *plane polarization* are synonyms.
- Light *polarization*, represented in waves by the direction of the electric field, is a property with a role in the processes of reflection and refraction, and is therefore in part responsible for the surface appearance of bodies. Therefore, it has an important role in colorimetry and should not be ignored, especially in colorimetric measurements, in which the radiation reflected by a body is measured and the optical components of the instrument reflect light.
- An example of linearly polarized light is the light emitted by an LCD monitor. It can be easily checked with Polaroid sun glasses rotated on the screen, which operates as a polarization analyser.
- In the majority of cases, the light is not polarized and it is not possible to define a single plane for the oscillation of the electric field, but thanks to the superposition principle, it is possible to consider it as a sum of linearly polarized waves. The decomposition of a light beam in waves of different polarization facilitates the discussion of the phenomena of reflection and refraction. It is also of practical utility since many everyday devices polarize light as, for example, polaroid filters used in sunglasses.
- The flow of power ϕ_e of the electromagnetic radiation (energy per unit of time and area, measured in W/m^2) proceeds in the direction orthogonal to the fields $\vec{E}$ and $\vec{H}$.
- The electric and magnetic field magnitudes are proportional to each other $|\vec{H}| = \sqrt{(\varepsilon / \mu)}|\vec{E}|$.
- The wave is *coherent*, which means when the phase is known at a point in space and at an instant in time, then it can be determined at any other point and instant.
- The velocity at which the plane waves propagate in space is implicitly defined by the argument of the wave functions $(\kappa x - \omega t - \vartheta_0)$, which takes the same value for different pairs of variables x and t. If we consider the wave function at the instants t_1 and t_2, it follows that this function has the same values at the points x_1 and x_2 such that $(\kappa x_1 - \omega t_1) = (\kappa x_2 - \omega t_2)$, from which $\omega (t_1 - t_2) = \kappa (x_1 - x_2)$, and then the

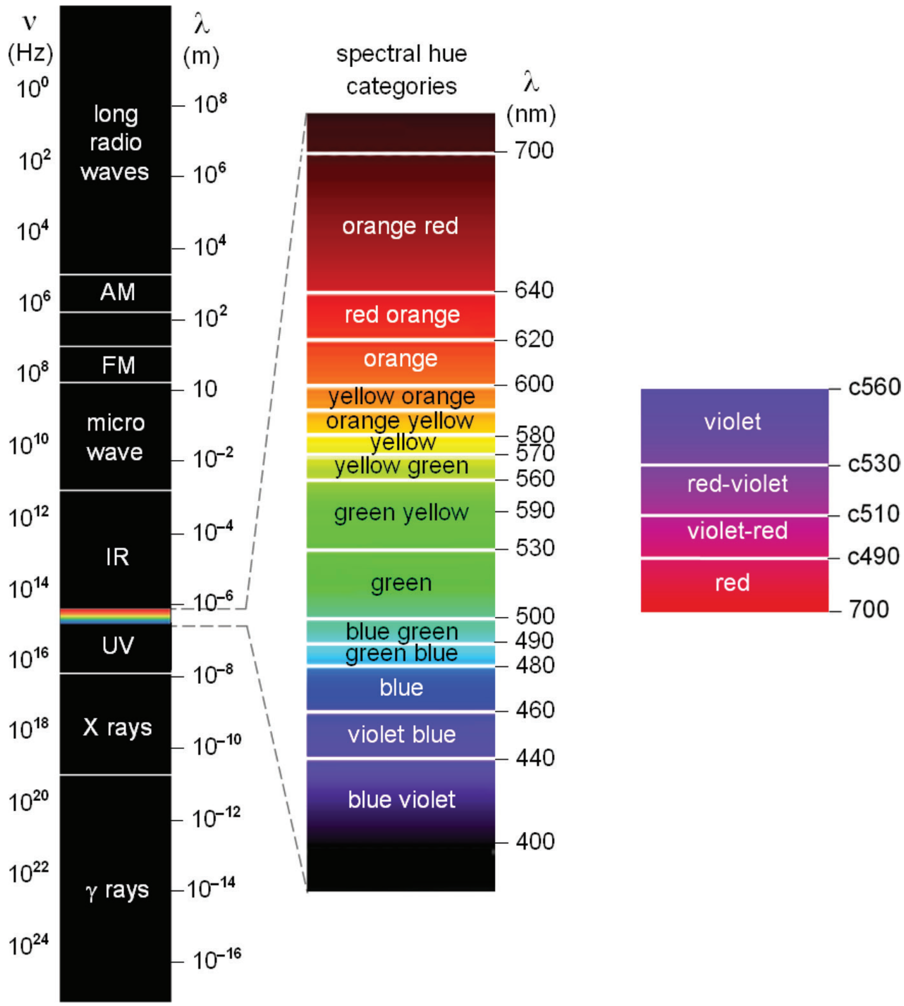

Figure 2.2 *Complete spectrum of electromagnetic radiation characterized by wavelength and frequency. The part of the spectrum related to visible light is expanded and a hue name is associated to each wavelength range. The colours printed here are only representative and approximate for many reasons, which will become clear on reading the book. For completeness, the non-spectral hues (purple and magenta hues) are added, which are specified by the complementary wavelengths (Section 4.3). 'c' before the wavelength numbers in the extra-spectral region means 'complementary of'.*

wave advances rigidly with a velocity $v = (x_1 - x_2)/(t_1 - t_2) = \omega/\kappa = \lambda/\tau = \lambda\nu$. The velocity in the vacuum is $c = 2.997925 \times 10^8$ m/s.

- The waves are classified by their wavelength λ or frequency $\nu = c/\lambda$. Since the velocity depends on the medium, by convention the wavelength is considered in a vacuum and is very close to that in air. A classification according to decreasing wavelength (Figure 2.2) distinguishes between radio waves (used in telecommunications), microwaves (typical of radar and microwave ovens), infrared radiation (IR), visible radiation (VIS), ultraviolet radiation (UV), X-rays and γ-rays. There are no precise limits for the spectral range of visible radiation since they depend upon the amount of radiant power reaching the retina and the responsivity of the photosensitive cells. Generally, the lower limit is taken between 360 and 400 nm and the upper limit between 760 and 830 nm. A finer subdivision of UV radiation distinguishes between UVC with $100 < \lambda < 280$ nm, UVB with $280 < \lambda < 315$ nm and UVA with $315 < \lambda < 400$ nm, while in IR radiation we have NIR (near-IR) with $0.8 < \lambda < 2.5$ μm, and IR-A with $0.78 < \lambda < 1.4$ μm, IR-B with $1.4 < \lambda < 3.0$ μm and IR-C with 3 μm $< \lambda <$ 1 mm.

"*Monochromatic radiation* – radiation characterized by a single frequency or a single wavelength.

NOTE 1 – In practice, radiation of a very small range of frequencies can be described by stating a single frequency or wavelength (NB: In reality a radiation with a single wavelength cannot exist).
NOTE 2 – The wavelength in air or in vacuum is also used to characterize a monochromatic radiation. The medium must be stated.
NOTE 3 – The wavelength in standard air is normally used in photometry, radiometry and colorimetry."[1–3]

A clarification is required: the radiation of a single wavelength is not selectable instrumentally (Section 12.3.1) and is theoretically definable only in an infinite optically homogeneous space. In a strict sense, the word 'monochromatic' is improperly used; anyway, it is used. This does not condition the science of colour, and the monochromatic radiations can be considered as a language approximation. A radiation, which is non-monochromatic but is defined within a narrow spectral range, is a *spectral radiation* defined within a band specified by its width (Section 12.3.1). In colorimetry, the bandwidth should be 1 nm (Section 6.10). In practice, often the spectral radiations used have a bandwidth up to 10 nm and Gaussian spectral distribution.

"*Monochromatic stimulus* – colour stimulus consisting of monochromatic radiation. Equivalent term: *spectral stimulus*."[1]

Anticipating the description of the colour sensations, Figure 2.2 proposes the visible electromagnetic radiation associated with the various spectral hues (albeit approximated by the printing process) and the corresponding names. There are extra-spectral hues provided by a mixture of radiation of short and long wavelengths with the intensity in a variable ratio. These extra-spectral hues cannot be associated with a wavelength; however, the convention is to associate them with the complementary wavelengths, denoted by the letter 'c' followed by the value of the complementary wavelength.

Complementary wavelength to a light radiation is the wavelength of the spectral radiation, which when mixed with this in an appropriate ratio of intensity generates a light-looking *achromatic* – without hue –.

The plane wave functions with different wavelengths are simultaneously solutions of Maxwell's equations because these differential equations are linear. It follows that also a linear combination of these waves is a wave function solution of Maxwell's equations. This is summarized in the *superposition principle*, so the sum of many solutions is still a solution. This latter property enables treating any light beam as a sum of monochromatic beams, which are defined by a single wavelength. This property is of particular importance in colour science.

Radiation defined by a superposition of different monochromatic waves is termed *heterochromatic*.