Ashish Tewari

Foundations of Space Dynamics

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FOUNDATIONS OF SPACE DYNAMICS

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FOUNDATIONS OF SPACE DYNAMICS

First Edition

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WILEY

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To the loving memory of my daughter, Manya (24.1.2000 - 9.7.2019)

Contents

Preface

Foundations of Space Dynamics is written as a textbook for students, as well as a ready reference covering the essential concepts for practicing engineers and researchers. It introduces a reader to the basic aspects of both orbital mechanics and attitude dynamics. While many good textbooks are available on orbital mechanics and attitude dynamics, there is a need for a direct, concise, yet rigorous treatment of both the topics in a single textbook. Important derivations from basic principles are highlighted, while offering insights into the physical principles which can often be hidden by mathematical details. While the emphasis is on analytical derivations, the essential computational tools are presented wherever required, such as the iterative root-finding methods and the numerical integration of ordinary differential equations.

The objective of this book is to provide a physically insightful presentation of space dynamics. The usage of simple ideas and numerical tools to illustrate advanced concepts is inspired by the work of the original masters (Newton, Liebnitz, Laplace, Gauss, etc.), and is combined with the application and terminology of modern space dynamics.

A student of space dynamics in the past generally possessed a strong background in analytical mechanics, often reinforced by such classical treatises as those by Whittaker, Lanczos, Truesdell, and Mach. Today, the exposure to analytical dynamics is often based upon a single undergraduate course. This book therefore includes a basic introduction to analytical mechanics by both Newtonian and Lagrangian approaches.

The contents of the textbook are arranged such that they may be covered in two successive courses: *Space Dynamics I* could focus on Chaps. 1–7 and 11, while the following course, *Space Dynamics II*, could cover Chaps. 8–10 and 12, supplemented by a semester project exploring a specific research topic. However, the arrangement of the chapters in the book offers sufficient flexibility for them to be covered in a single comprehensive course, if so required. There are a multitude of exercises at the end of the chapters which can serve as homework assignments and quiz problems. Solutions to selected exercises is also provided.

I would like to thank the editorial and production staff of Wiley, Chichester, for their constructive suggestions and valuable insights during the preparation of the manuscript.

Ashish Tewari
May 2020

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Introduction

This chapter gives an introduction to the basic features of space flight, which is predominated by the quiet space environment and gravity. The essential differences with atmospheric flight are discussed, and the important time scales and frames of reference for space flight are described. Topics in space dynamics are classified as the translational motion (orbital mechanics) and rotational motion (attitude dynamics) of a rigid spacecraft. Classification of the various practical spacecraft is given according to their missions.

1.1 Space Flight

Space flight refers to motion outside the confines of a planetary atmosphere. It is different from atmospheric flight in that no assistance can be derived from the atmospheric forces to support a vehicle, and no benefit of planetary oxygen can be utilized for propulsion. Apart from these major disadvantages, space flight has the advantage of experiencing no (or little) drag due to the resistance of the atmosphere; hence a spacecraft can achieve a much higher flight velocity than an aircraft. Since atmospheric lift is absent to sustain space flight, a spacecraft requires such high velocities to balance the force of gravity by a centrifugal force in order to remain in flight. The trajectories of spacecraft (called *orbits*) – being governed solely by gravity – are thus much better defined than those of aircraft. Since gravity is a conservative force, space flight involves a conservation of the sum of kinetic and potential energies, as well as that of the angular momentum about a fixed point. Therefore, space flight is much easier to analyze mathematically when compared to atmospheric flight.

1.1.1 Atmosphere as Perturbing Environment

When can the effects of the atmosphere be considered negligible so that space flight can come into existence? The atmosphere of a planetary body – being bound by gravity – becomes less dense as the distance from the planetary surface (called *altitude*) increases, owing to the inverse-square diminishing of the acceleration due to gravity from the planetary centre. For an atmosphere completely at rest, this relationship between the atmospheric density,

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ρ, and the altitude, z, can be derived from the following differential equation of *aerostatic equilibrium* (Tewari, 2006):

$$dp = -\rho g dz \tag{1.1}$$

where p refers to the atmospheric pressure, and g the *acceleration due to gravity* prevailing at a given altitude. For a spherical body of radius r_0, the gravity obeys the inverse-square law discovered by Newton, given by

$$g = g_0 \frac{r_0^2}{(r_0 + z)^2} \tag{1.2}$$

where g_0 is the acceleration due to gravity at the surface of the body (i.e., at $z = 0$). When Eq. (1.2) is substituted into Eq. (1.1), and the thermodynamic properties of the atmospheric gases are taken into account, the differential equation, Eq. (1.1), can be integrated to yield an algebraic relationship between the *atmospheric density*, ρ, and the altitude, z, called an atmospheric model. For Earth's atmosphere, one such model is the *U.S. Standard Atmosphere 1976* (Tewari, 2006), whose predicted density variation with the altitude in the range $0 \leq z \leq 250$ km is listed in Table 1.1. It is evident from Table 1.1 that the atmospheric density, ρ, can be considered to be negligible for a flight for $z \geq 120$ km around Earth. A similar (albeit smaller) value of ρ is obtained on Mars at $z = 120$ km. Hence, for both Earth and Mars, $z = 120$ km can be taken to be the boundary above which the *space* begins.

The flight of a spacecraft around a large spherical body of radius r_0 is assumed to take place outside the atmosphere, (such as $z > 120$ km for Earth and Mars), and is governed by the gravity of the body, with acceleration given by Eq. (1.2). Space-flight trajectories are well defined *orbits*

Table 1.1 Variation of density with altitude in Earth's atmosphere

Altitude, z (km)	Density, ρ kg/m^3
0	1.2252
1	1.1119
5	0.7366
10	0.4136
20	0.0891
30	0.0185
40	0.0041
50	0.0011
60	3.24×10^{-4}
70	8.65×10^{-5}
80	2.04×10^{-5}
90	3.90×10^{-6}
100	6.94×10^{-7}
110	1.37×10^{-7}
120	3.40×10^{-8}
150	2.57×10^{-9}
200	4.66×10^{-10}
250	1.41×10^{-10}

due to the simple nature of Eq. (1.2). However, since the atmospheric density in a very low orbit (e.g., $120 < z < 250$ km on Earth), albeit quite small, is not exactly zero, the flight of a spacecraft can be gradually affected, to cause significant deviations over a long period of time from the orbits predicted by Eq. (1.2). This is due to the fact that the atmospheric forces and moments are directly proportional to the flight dynamic pressure, $1/2\rho v^2$, where v is the flight speed. The high orbital speed, v, required for space flight makes the dynamic pressure appreciable, even though the density, ρ, is by itself negligible. The atmospheric *drag* (the force resisting the motion) causes a slow but steady decline in the flight speed, until the latter falls below the magnitude where an orbital motion can be sustained. Thus atmospheric drag can cause a low-orbiting satellite to slightly decay in altitude after every orbit, and to ultimately enter the lower (dense) portions of the atmosphere, where the mechanical stress created by the ever increasing dynamic pressure, as well as the heat generated by atmospheric friction, lead to its destruction. Therefore, for predicting the life of a satellite in a low orbit, the atmospheric effects must be properly taken into account. Figure 1.1 shows an example of the decay in the orbit of a spacecraft initially placed into a circular orbit of $z = 200$ km around Earth. In this simulation obtained by a *Runge-Kutta method* (Appendix A), the spacecraft is assumed to be a sphere of 1 m diameter, with a constant free-molecular drag coefficient of 2.0 (Tewari, 2006). As seen in the figure, the altitude decays quite rapidly as the number of orbits, N, increases. The initial average rate of altitude loss seen in Fig. 1.1 – 1 km per 4 orbits – is likely to increase as the spacecraft descends lower, thereby encountering a higher density. When the spacecraft is placed in a circular orbit of $z = 180$ km, its altitude decays very rapidly, and it re-enters the

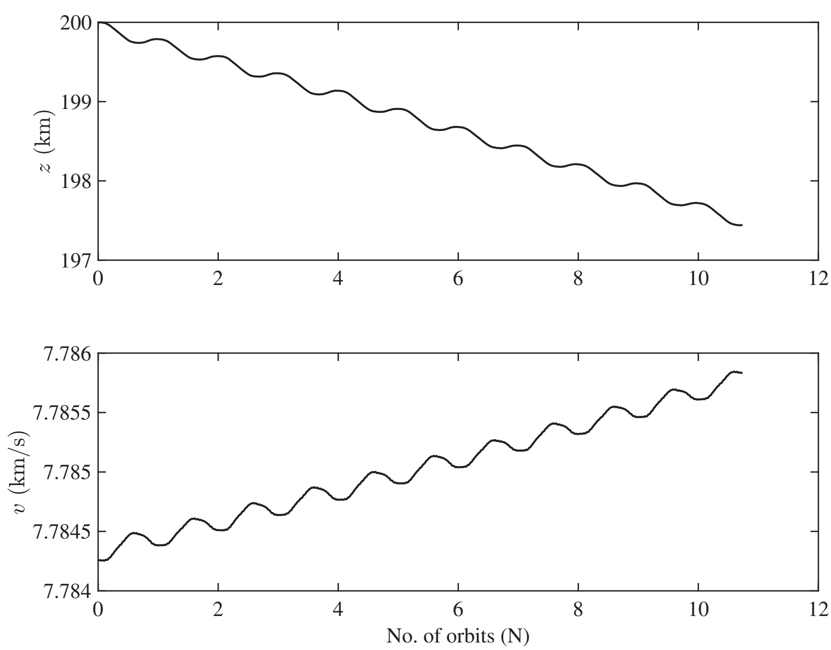

Figure 1.1 Decay in the orbit due to atmospheric drag for a spacecraft initially placed in a circular orbit of $z = 200$ km around Earth.

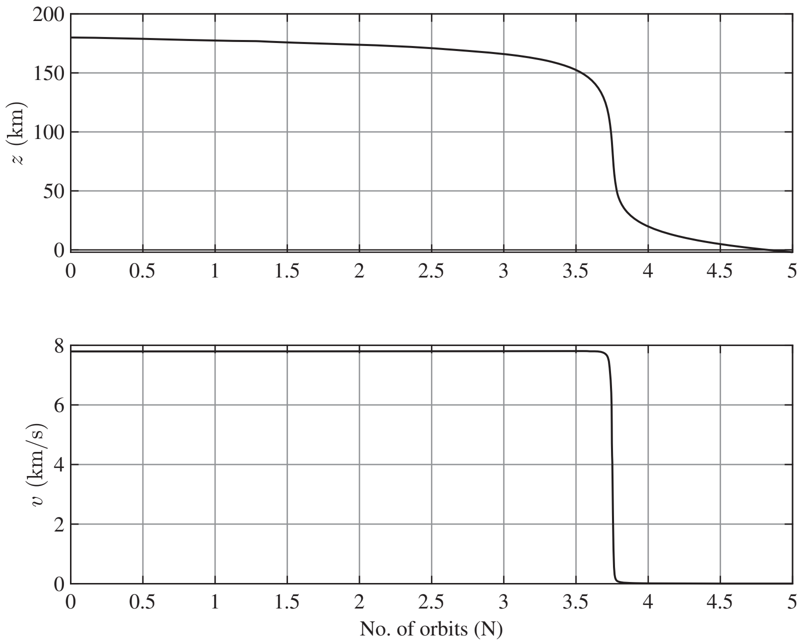

Figure 1.2 Decay in the orbit due to atmospheric drag for a spacecraft initially placed in a circular orbit of $z = 180$ km around Earth.

atmosphere after only 3.5 orbits (Fig. 1.2). Hence, the life of the spacecraft is only about 3.5 revolutions in a circular orbit of altitude 180 km above Earth. As Figs. 1.1 and 1.2 indicate, a stable orbit around Earth for this spacecraft should have $z > 200$ km at all times.

Apart from the atmospheric effects, there are other environmental perturbations to a spacecraft's flight around a central body, which is assumed to be spherical as required by Eq. (1.2). These are the gravity of the actual (non-spherical) shape of the central body, as well as the gravity of other remote large bodies, and the solar radiation pressure. However, such effects are typically small enough to be considered small perturbations when compared to the spherical gravity field of the central body given by Eq. (1.2). Such effects can be regarded as small perturbations applied to the orbit governed by Eq. (1.2), and should be carefully modelled in order to predict the actual motion of the spacecraft.

1.1.2 Gravity as the Governing Force

Space flight is primarily governed by gravity. "Governing" implies dictating the path a given body describes in a three-dimensional space. Aircraft and rocket flights are *not* primarily governed by gravity, because there are other forces acting on the body, such as the lift and the thrust, which are of comparable magnitudes to that of gravity and therefore determine the flight path. Discovered and properly analyzed for the first time by Newton in the late 17$^{\text{th}}$ century, gravity can be expressed simply, but has profound consequences. For example, by applying Newton's law of gravitation, it could have been inferred that the universe cannot be static, because gravity would cause all the objects to collapse towards a single point. However, this simple fact escaped the notice of all physicists ranging from Newton himself to Einstein, until it was observed by Hubble in 1924 that the universe is expanding at a rate which increases with the distance between any two objects. A reader may be cautioned against the complacency which often arises by treating the motion governed by gravity as simple (even trivial) to understand. There are many surprising and interesting consequences of gravity being the governing force in flight, such as Kepler's third law of planetary motion, which implies that the time period of an orbiting body depends only upon the mean radius, and is independent of the shape of the orbit. A larger part of a course on space dynamics involves understanding gravity and its effects on the motion of a body in space.

1.1.3 Topics in Space Dynamics

Space dynamics consists of two parts: (a) *orbital mechanics*, which describes the translation in space of the centre of mass of a rigid body primarily under the influence of gravity, and (b) *attitude dynamics*, which is the description of the rotation of the rigid body about its own centre of mass. While these two topics are largely studied separately, in some cases orbital mechanics and attitude dynamics are intrinsically coupled, such as when the rigid body experiences an appreciable gravity-gradient torque during its orbit. Furthermore, when designing an attitude control system for a spacecraft, it is necessary to account for its orbital motion. Therefore, while elements of orbital mechanics and attitude dynamics can be grasped separately, their practical application involves a combined approach.

1.2 Reference Frames and Time Scales

Space flight requires a definite background of objects to measure distances, as well as to orient the spacecraft in specific directions. Since fixed objects are hard to come by in practice, navigation and attitude determination are non-trivial problems in space flight. Such a problem does not exist for the motion taking place on, or very close to, a solid surface, where ground-fixed objects can serve as useful references for both navigation and orientation of the vehicles.

1.2.1 Sidereal Frame

Three mutually perpendicular straight lines joining distant objects constitute a *reference frame*. Generally, distant objects in the universe are moving with respect to one another; hence the straight lines joining them would rotate, as well as either stretch out or contract with time. Suppose one can find two objects which are fixed relative to each other. Then a straight line joining them would be fixed in length, and a vector pointing from one object to the other would always have a constant direction. A reference frame consisting of axes which have fixed directions is said to be a *sidereal frame*. There are certain directions which can be used to orient a sidereal frame. For example, the orbital plane of Earth around the sun, called the *ecliptic*, intersects Earth's equatorial plane along a straight line called the *line of nodes*. The *nodes* are the two specific points where this line intersects Earth's orbit, as shown in Fig. 1.3. One of the two

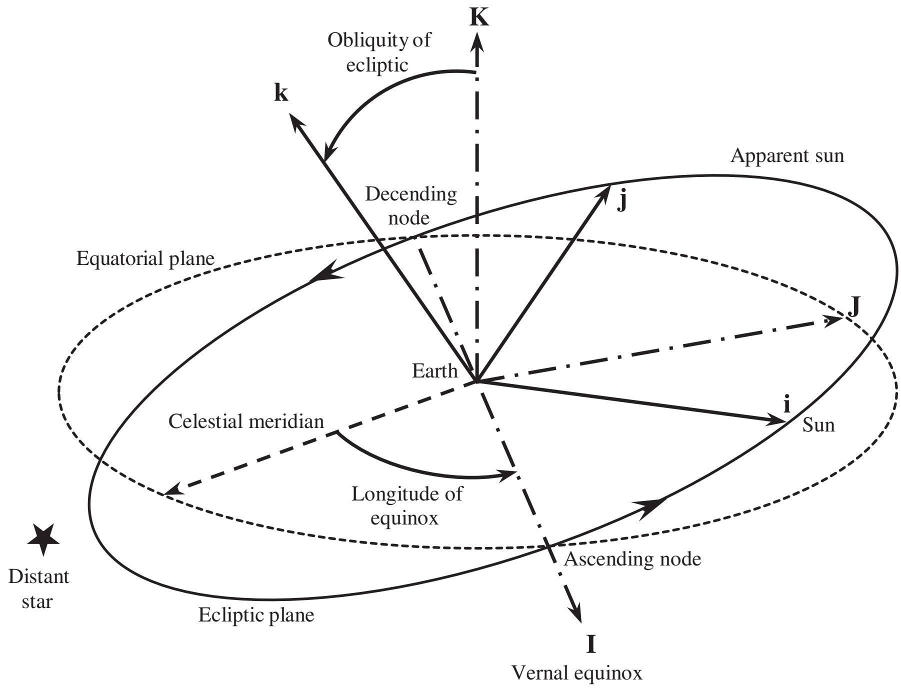

Figure 1.3 The equinoctial sidereal frame (**I**, **J**, **K**), the ecliptic synodic frame (**i**, **j**, **k**), and Earth centred celestial meridian.

nodes is an *ascending node*, where the apparent motion of the sun as seen from Earth (called the *apparent Sun*) occurs from the south to the north of the equator. This happens at the *vernal equinox*, occurring every year around March 21. The *descending node* of the apparent sun is at the *autumnal equinox*, which takes place around September 22. Since the vernal equinox points in a specific direction from the centre of Earth, it can be used to orient one of the axes of the sidereal frame, as the axis **I** in Fig. 1.3. Another axis of the sidereal frame can be taken to be normal to either the ecliptic or the equatorial plane (axis **K** in Fig. 1.3), and the third axis can be chosen to be perpendicular to the first two (axis **J** in Fig. 1.3).

The rate of rotation of Earth on its own axis (normal to the equatorial plane) is from the west to the east, and can be measured in a sidereal reference frame oriented with the vernal equinox direction. This rate is called the *sidereal rotation rate*, and would be the true rotation rate of Earth if the vernal equinox were a constant direction. A *sidereal day* is the period of rotation of Earth measured from the vernal equinox. If the sun is used for timing the rotational rate of Earth, the period from noon to noon is a *mean solar day* (m.s.d.) of 24-hour duration. However, the mean solar day is not the true rotational rate of Earth because of Earth's orbit around the sun, which also takes place from the west to the east. To calculate the sidereal day from the mean solar day, a correction must be applied by adding the average rate at which Earth orbits the sun. The *tropical year* is the period of Earth's orbit around the sun measured from one vernal equinox to the next, and equals 365.242 mean solar days. This implies that the mean apparent sun is slightly less than one degree per day ($360^\circ/365.242$). Such a correction gives the sidereal day as the following:

$$\frac{1}{\frac{1}{24\times3600}+\frac{1}{365.242\times24\times3600}} = 86164.0904\ \mathrm{s}\,, \tag{1.3}$$

or 23 hr., 56 min., 4.0904 s.

Unfortunately, the vernal equinox is not a constant direction because of the slow *precession* of Earth's axis (thus the equatorial plane) caused by the gravitational influence of the sun and the moon (called the *luni-solar attraction*). When a spinning rigid body, such as Earth, is acted upon by an external torque, such as due to the gravity of the sun and the moon, its spin axis undergoes a complex rotation called "precession" and "nutation", which will be explained in detail in Chapter 11. This rotation of the equatorial plane causes the two equinoxes to shift towards the west, and is thus called the *precession of the equinoxes*. The period of the precession is about 25772 yr., which implies that the sidereal day differs only slightly from the true rotational period of Earth. It also means that an equinoctial sidereal reference frame, such as the frame $(\mathbf{I}, \mathbf{J}, \mathbf{K})$ in Fig. 1.3, rotates very slowly against a background of distant stars. Hence the vernal equinox (and the equinoctial sidereal reference frame) can be approximated to be the fixed references for most space flight applications. However, for a long flight time of several years' duration, the calculations must be brought to a common reference at a specific time (called an *epoch*[1]) by applying the necessary corrections, which take into account the slow movement of the vernal equinox towards the west. The equinox is given for various epochs by the *International Earth Rotation and Reference Systems Service* (IERS) in terms of the longitude of the equinox measured from a *celestial meridian* (see Fig. 1.3). The inclination of Earth's spin axis from the normal to the ecliptic is called the *obliquity of the ecliptic* (Fig. 1.3), and also varies with time due to the *nutation* caused by the luni-solar attraction. (The precession and nutation, discussed in detail in Chapter 11, cause Earth's spin axis to rotate with time due to the luni-solar attraction.) The value of the obliquity of the ecliptic in the current epoch is measured by IERS to be about 23°26'21". The period of nutation of Earth's spin axis is about 41000 yr., which is considerably longer than the period of its precession. The precession and nutation are explained in Chapter 11 when considering the rotation of a rigid body (such as Earth).

Apart from the precession and the nutation of Earth's spin axis, there is also a precession of the ecliptic caused by the gravitational attraction of the other planets. This is a much smaller variation in the equinoxes (about 100 times smaller than that caused by luni-solar attraction).

Since the vernal equinox moves slightly westward every year, the tropical year is not the true period of revolution of Earth in its orbit around the sun. The true period of revolution is the *sidereal year*, which is measured by timing the passage of Earth against the background of distant stars, and equals 365.25636 mean solar days. Thus a tropical year is shorter than the actual year by 20 hr., 40 min., and 42.24 s.

[1] An epoch is a moment in time used as a reference point for a time-varying astronomical quantity, such as the orbital elements specifying the shape and the plane of an orbit, the direction of the spin axis of a body, the coordinates of important celestial objects, etc.

1.2.2 Celestial Frame

For a motion taking place inside the solar system, any two stars (except the sun) appear to be fixed for the duration of the motion. Hence, a reference frame constructed out of three mutually perpendicular axes, each of which are pointing towards different distant stars, would appear to be fixed in space, and can serve as a sidereal reference frame. A reference frame fixed relative to distant stars is termed a *celestial reference frame*. For example, the rate of rotation of Earth about its own axis can be measured by an observer standing astride the North Pole by timing the rate at which a straight line joining Earth to a distant star, called a celestial meridian (see Fig. 1.3), appears to rotate. This rate gives the true rotational time period of Earth, called the *stellar day*, which is measured by IERS to be 23 hr., 56 min., 4.0989 s. Hence, the sidereal day is shorter than the stellar day by about 8.5×10^{-3} s.

1.2.3 Synodic Frame

When two objects orbit one another at nearly constant rates on a fixed plane, a reference frame can be defined by two of its axes on the plane of rotation and rotating at the constant rate, and the third axis normal to the plane. Such a rotating reference frame is called a *synodic frame*. An example of a synodic frame is the *ecliptic frame*, which is a reference frame constructed out of the ecliptic plane, such as the frame $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ in Fig. 1.3. The motion of an object measured relative to a synodic frame must be corrected by a vector subtraction of the motion of the frame itself, as exemplified by the calculation of the sidereal day from the observed rotation in the ecliptic frame. The ecliptic frame has been used as a reference since the earliest days of astronomical observations. The division of the circle into 360° arose out of the apparent motion of the sun per day, which subtends an arc of one diameter every 12 hours when seen from Earth. Since the moon's apparent diameter from Earth is roughly the same as that of the sun, the eclipses of the sun and the moon are observed in the ecliptic (thus the name). However, since the moon's orbital plane around Earth is tilted $\pm 5.1°$ relative to the ecliptic, the eclipses happen only along the intersection (i.e., the line of nodes) of the two planes.

The Earth-moon line provides another synodic reference frame for space flight. The Earth and the moon describe coplanar circles about the common centre of mass (called the *barycentre*) every 27.32 mean solar days relative to the vernal equinox (called a *sidereal month*). This rotational period appears in the synodic frame to be 29.53 mean solar days (a *synodic month*) from one new moon to the next, which is obtained from the sidereal month by subtracting the rate of revolution of Earth-moon system around the sun.

1.2.4 Julian Date

Instead of the *calendar year* of 365 mean solar days, the tropical year of 365.242 mean solar days, and the sidereal year of 365.25636 mean solar days, it is much more convenient to use a *Julian year* of 365.25 mean solar days, which avoids the addition of leap years in carrying out astronomical calculations. A *Julian day number* (*JDN*) is defined to be the continuous count of the number of mean solar days elapsed since 12:00 noon *universal time* (UT) on January 1, 4713 BC. Universal time refers to the time taken as 12:00 noon when the sun is directly over the Greenwich meridian (which is defined to be zero longitude). The Julian day number 0 is assigned to the day starting at that time on the Julian proleptic calendar. The *Julian date* of a general time instant is expressed as the JDN plus the fraction of the 24-hour day elapsed since the preceding noon UT. Julian dates are thus expressed as a Julian day number plus a decimal fraction. For example, the Julian date for 10:00 a.m. UT on April 21, 2020, is given by J2458960.91667, and the JDN is 2458960. Epochs are listed in ephemeris charts and nautical almanacs according to their Julian dates. Hence a Julian date serves as a common time measure for astronautical calculations involving two events separated in time.

Computation of the Julian date (JD) from a Gregorian calendar date is complicated due to the three calendar cycles used to produce the Julian calendar, namely the solar, the lunar, and the indiction cycles of 28, 19, and 15 year periods, respectively (Seidelmann, 1992). A product of these gives the *Julian period* of 7980 years. The Julian period begins from 4713 BC, which is chosen to be the first year of solar, lunar, and indiction cycles beginning together. The next epoch when the three cycles begin together will happen at noon UT on January 1, 3268. The following conversion formula for the JDN, truncated to the last integer, uses the numbering of the months from January to December as $M = 1, 2, \ldots, 12$; the Gregorian calendar years are numbered such that the year 1 BC is the year zero, $Y = 0$, (i.e., 2 BC is $Y = -1$, 4713 BC is $Y = -4712$, etc.); and the day number, D, is the last completed day of the month up to noon UT:

$$\begin{aligned} JDN &= D + \frac{1461}{4}\left(Y + 4800 + \frac{M-14}{12}\right) \\ &\quad - \frac{3}{400}\left(Y + 4900 + \frac{M-14}{12}\right) - 31708\,. \end{aligned} \tag{1.4}$$

This formula calculates the JDN for 09:25 a.m. UT on June 25, 1975, by taking $Y = 1975$, $M = 6$, $D = 24$, and yields the last truncated integer value as $JDN = 2442589$. Then the time elapsed from noon UT on June 24 to 09:25 a.m. UT on June 25 is added as a fraction to give the following Julian date:

$$JD = JDN + \frac{12 + 9 + 25/60}{24} = 2442589.892361\,.$$

An epoch in the Julian date is designated with the prefix J, and the suffix being the closest Gregorian calendar date. For example, $J2000$ refers to 12:00 noon UT on January 1, 2000, and has the Julian date of 2451545. Similarly, the epoch $J1900$, which occurs exactly 100 Julian years *before* 12:00 noon UT on January 1, 2000, must refer to 12 noon UT on January 0, 1900; hence its date in the Gregorian calendar is December 31, 1899, and its Julian date is 2415020. The difference in the epochs $J2000$ and $J1900$ is therefore $2451545 - 2415020 = 36525$ mean solar days (which is exactly 100 Julian years).

Since Julian day numbers with the epoch $-J4712$ can become very large, it is often convenient to use a later epoch for computing JD. Epochs can be chosen with simpler JDN figures, such as 12:00 hr. UT on November 16, 1858, which has $JDN = 2400000$. Then Julian dates can be converted to this epoch by replacing JD with $JD - 2400000$. For example, the Julian date for 09:25 a.m. UT, June 25, 1975, converted to the epoch of Nov. 16, 1858, is $JD = 42589.892361$. For the consistency of data, all modern astronomical calculations are reduced to the epoch, $J2000$, by international agreement. This means that all the Julian dates must be converted to this epoch by replacing JD with $JD - 2451545$.

1.3 Classification of Space Missions

Spacecraft are classified according to their missions. A large majority of spacecraft orbit Earth as artificial satellites for observation, mapping, thermal and radio imaging, navigation, scientific experimentation, and telecommunications purposes. These satellites are classified according to the shapes and sizes of their orbits. A spacecraft orbiting a central body at altitudes smaller than the mean radius, r_0, of the body, $z < r_0$, is termed a *low-orbiting spacecraft*. Examples of such spacecraft for Earth ($r_0 = 6378.14$ km) are the *low-Earth orbit* (LEO) satellites, which orbit the planet in nearly circular orbits of $200 \le z \le 2000$ km. Orbital periods of LEO satellites range from 90 to 127 min., and are mainly used for Earth observation, photo reconnaissance, resource mapping, and special sensing and scientific missions. The *International Space Station* is a manned LEO spacecraft with a nearly circular orbit of mean altitude, $z = 400$ km. There are hundreds of active LEO satellites in orbit at any given time, launched by various nations for civil and military applications.

A *medium-Earth orbit* (MEO) satellite has a period of about 12 hours. Examples of such spacecraft are the *Global Positioning System* (GPS) navigational satellites in circular orbits of altitudes about 20,000 km, and *Molniya* telecommunications satellites of Russia in highly eccentric elliptical orbits inclined at 63.435° relative to Earth's equatorial plane.

The highest altitude of Earth satellites is for those in the *geosynchronous equatorial orbit* (GEO), which is a circular orbit in the equatorial plane of a period exactly matching a sidereal day, i.e., 23 hr., 56 min., 4.0904 s. This translates into an altitude of $z = 35786.03$ km. Since the orbital frequency of a GEO satellite equals the rate of rotation of Earth on its axis, such a satellite returns to the same point above the equator after each sidereal day, thereby appearing to be stationary to an observer on the ground. Hence, a GEO satellite is used as a telecommunications relay platform for signals between any two ground stations directly in the line of sight of the satellite. Due to the high altitude of the GEO satellite, a broad coverage of signals is provided to the receiving stations on the ground, and is the basis of modern television broadcasts and mobile telephone communications.

A small number of spacecraft are put into highly specialized lunar, interplanetary, and asteroid/cometary intercept orbits for the exploration of the solar system. Due to the typically large distances involved in their missions, which might include the time spent beyond the line-of-sight of Earth, such spacecraft must be fully autonomous in terms of their basic operations. The spacecraft which are sent to explore the outer planets (such as NASA's *Voyager 1* and *Voyager 2*, *Cassini*, *Galileo*, and *New Horizons*) must also have an onboard electrical power source for charging their batteries, due to the unavailability of effective solar power (the sun is too dim at such large distances).

Exercises

1. Using the following exponential atmosphere model for Earth with the scale height, $H = 6.7$ km, and base density, $\rho_0 = 1.752\ \mathrm{kg/m^3}$, calculate the atmospheric density at the altitude, $z = 150$ km:

$$\rho = \rho_0 e^{-z/H}$$

   Compare the result with that given in Table 1.1.

2. Calculate the Julian date for 3:30 p.m. UT on October 15, 2007, referring to the $J2000$ epoch.

3. What is the exact time difference between two events happening at 11:05 a.m. on July 28, 1993, and 8:31 p.m. on November 3, 2005, respectively?

References

Tewari A 2006. *Atmospheric and Space Flight Dynamics*. Birkhäuser, Boston.

Seidelmann KP (ed.) 1992. *Explanatory Supplement to the Astronomical Almanac*. University Science Books, Sausalito, CA.

2

Dynamics

Dynamics is the study of an object in motion, and pertains to a change in the position and orientation of the object as a function of time. This chapter introduces the basic principles of dynamics, which are later applied to the motion of a vehicle in the space.

2.1 Notation and Basics

The vectors and matrices are denoted throughout this book in boldface, whereas scalar quantities are indicated in normal font. The elements of each vector are arranged in a column. The *Euclidean norm* (or *magnitude*) of a three-dimensional vector, $\mathbf{A} = (A_x, A_y, A_z)^T$, is denoted as follows:

$$| \mathbf{A} |= A = \sqrt{A_x^2 + A_y^2 + A_z^2}\,. \tag{2.1}$$

All the variables representing the motion of a spacecraft are changing with *time*, t. The overdots represent the time derivatives, e.g., $\mathrm{d}x/\mathrm{d}t = \dot{x}$, $\dot{A} = \mathrm{d}A/\mathrm{d}t$, $\ddot{A} = \mathrm{d}^2A/\mathrm{d}t^2$. The time derivative of a vector $\mathbf{A}$, which is changing both in its magnitude and its direction, requires an explanation.

The time derivative of a vector, $\mathbf{A}$, which is changing both in magnitude and direction can be resolved in two mutually perpendicular directions – one along the original direction of $\mathbf{A}$, and the other normal to it on the plane of the rotation of $\mathbf{A}$. The instantaneous *angular velocity*, $\boldsymbol{\omega}$, of $\mathbf{A}$ denotes the vector rate of change in the direction, whereas $\dot{A}$ is the rate of change in its magnitude. By definition, $\boldsymbol{\omega} \times \mathbf{A}$ is normal to the direction of the unit vector, $\mathbf{A}/A$, and lies in the instantaneous plane of rotation normal to $\boldsymbol{\omega}$. The rotation of $\mathbf{A}$ is indicated by the *right-hand rule*, where the thumb points along $\boldsymbol{\omega}$, and the curled fingers show the instantaneous direction

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of rotation,[1] $\boldsymbol{\omega}\times\mathbf{A}$. The time derivative of $\mathbf{A}$ is therefore expressed as follows:

$$\dot{\mathbf{A}} = \frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \dot{A}\frac{\mathbf{A}}{A} + \boldsymbol{\omega}\times\mathbf{A}\,, \tag{2.2}$$

where the term $\mathbf{A}/A$ represents a unit vector in the original direction of $\mathbf{A}$, and $\boldsymbol{\omega}\times\mathbf{A}$ is the change normal to $\mathbf{A}$ caused by its rotation. Equation (2.2) will be referred to as the *chain rule* of vector differentiation in this book.

Similarly, the second time derivative of $\mathbf{A}$ is given by the application of the chain rule to differentiate $\dot{\mathbf{A}}$ as follows:

$$\begin{aligned}
\ddot{\mathbf{A}} = \frac{\mathrm{d}^2\mathbf{A}}{\mathrm{d}t^2} = \frac{\mathrm{d}\dot{\mathbf{A}}}{\mathrm{d}t} &= \frac{\mathrm{d}(\dot{A}/A)}{\mathrm{d}t}\mathbf{A} + \left(\frac{\dot{A}}{A}\right)\dot{\mathbf{A}} + \dot{\boldsymbol{\omega}}\times\mathbf{A} + \boldsymbol{\omega}\times\dot{\mathbf{A}} \\
&= \left(\frac{A\ddot{A}-\dot{A}^2}{A^2}\right)\mathbf{A} + \left(\frac{\dot{A}}{A}\right)\left(\dot{A}\frac{\mathbf{A}}{A} + \boldsymbol{\omega}\times\mathbf{A}\right) + \dot{\boldsymbol{\omega}}\times\mathbf{A} + \boldsymbol{\omega}\times\left(\dot{A}\frac{\mathbf{A}}{A} + \boldsymbol{\omega}\times\mathbf{A}\right) \\
&= \ddot{A}\left(\frac{\mathbf{A}}{A}\right) + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{A}) + 2\boldsymbol{\omega}\times\left(\frac{\dot{A}}{A}\right)\mathbf{A} + \dot{\boldsymbol{\omega}}\times\mathbf{A}\,.
\end{aligned} \tag{2.3}$$

Applying Eq. (2.1) to the time derivative of the angular velocity, $\boldsymbol{\omega}$, we have the following expression for the *angular acceleration* of $\mathbf{A}$:

$$\dot{\boldsymbol{\omega}} = \dot{\omega}\left(\frac{\boldsymbol{\omega}}{\omega}\right) + \boldsymbol{\Omega}\times\boldsymbol{\omega}\,, \tag{2.4}$$

where $\boldsymbol{\Omega}$ is the instantaneous angular velocity at which the vector $\boldsymbol{\omega}$ is changing its direction. Hence, the second time derivative of $\mathbf{A}$ is expressed as follows:

$$\ddot{\mathbf{A}} = \left[\ddot{A}\left(\frac{\mathbf{A}}{A}\right) + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{A})\right] + \left(\frac{2\dot{A}}{A} + \frac{\dot{\omega}}{\omega}\right)\boldsymbol{\omega}\times\mathbf{A} + (\boldsymbol{\Omega}\times\boldsymbol{\omega})\times\mathbf{A}\,. \tag{2.5}$$

The bracketed term on the right-hand side of Eq. (2.5) is parallel to $\mathbf{A}$, while the second term on the right-hand side is perpendicular to both $\mathbf{A}$ and $\boldsymbol{\omega}$. The last term on the right-hand side of Eq. (2.5) denotes the effect of a time-varying axis of rotation of $\mathbf{A}$.

2.2 Plane Kinematics

As a special case, consider the motion of a point, P, in a fixed plane described by the *radius* vector, $\mathbf{r}$, which is changing in time. The vector $\mathbf{r}$ is drawn from a fixed point, o, on the plane, to the moving point, P, and hence denotes the instantaneous radius of the moving point from o. The instantaneous rotation of the vector $\mathbf{r}$ is described by the angular velocity, $\boldsymbol{\omega} = \omega\mathbf{k}$, which is fixed in the direction given by the unit vector $\mathbf{k}$, normal to the plane of motion. Thus we have the following in Eq. (2.4):

$$\boldsymbol{\Omega} = 0\,.$$

[1] A reference frame, $(\mathbf{i},\mathbf{j},\mathbf{k})$, consisting of three mutually perpendicular axes, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, is termed a *right-handed frame* if it satisfies the right-hand rule of vector multiplication of the first two axes in the proper sequence, to produce the third axis:

$$\mathbf{i}\times\mathbf{j} = \mathbf{k}\,.$$