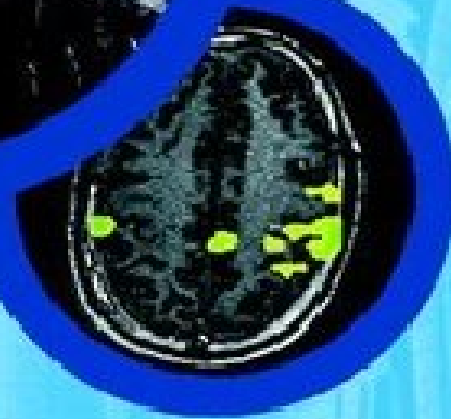


# Nuclear Physics

Principles  
and Applications

John Lilley



 WILEY

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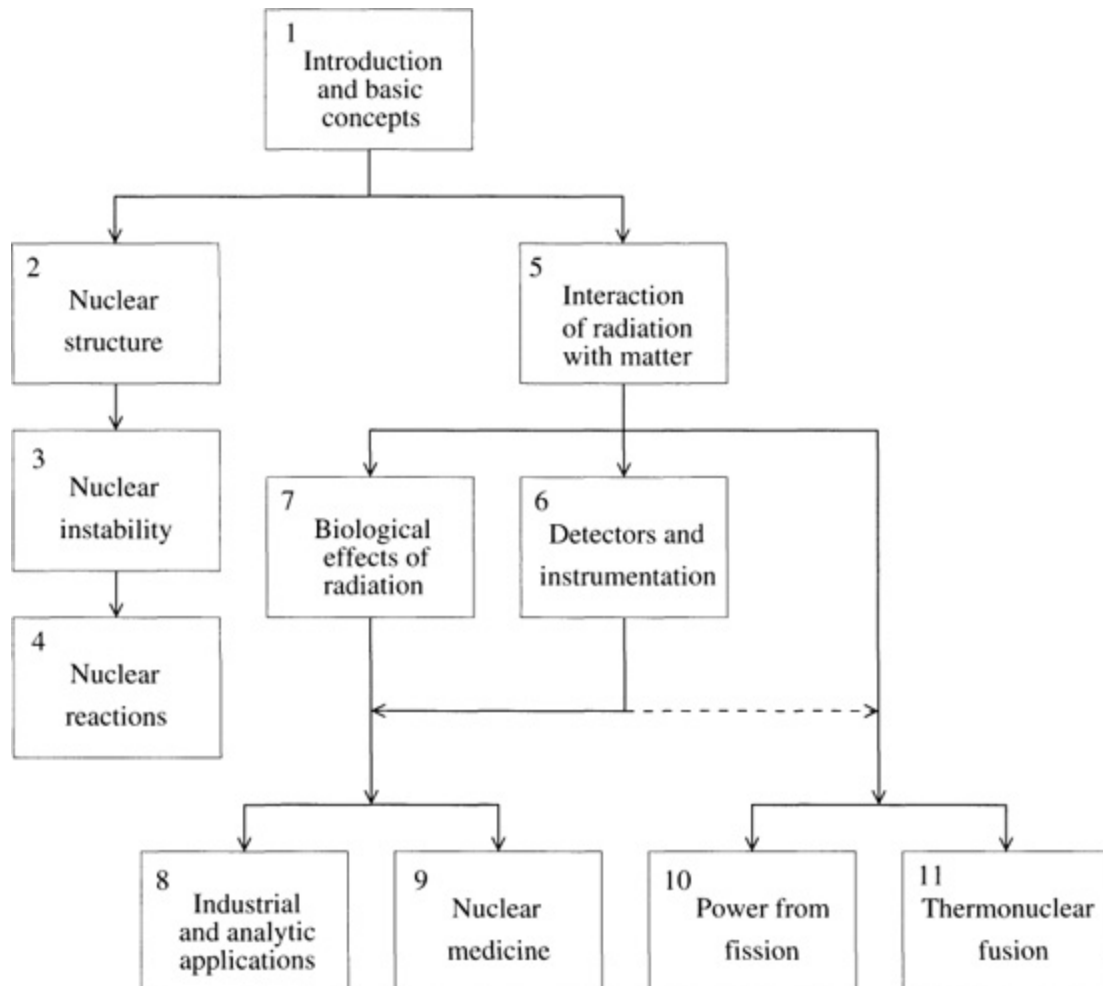
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# Flow diagram



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# **Nuclear Physics**

Principles and Applications

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**J. S. Lilley**

*Department of Physics and Astronomy  
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# Editors' preface to the Manchester Physics Series

The Manchester Physics Series is a series of textbooks at first degree level. It grew out of our experience at the Department of Physics and Astronomy at Manchester University, widely shared elsewhere, that many textbooks contain much more material than can be accommodated in a typical undergraduate course; and that this material is only rarely so arranged as to allow the definition of a short self-contained course. In planning these books we have had two objectives. One was to produce short books: so that lecturers should find them attractive for undergraduate courses; so that students should not be frightened off by their encyclopaedic size or price. To achieve this, we have been very selective in the choice of topics, with the emphasis on the basic physics together with some instructive, stimulating and useful applications. Our second objective was to produce books which allow courses of different lengths and difficulty to be selected with emphasis on different applications. To achieve such flexibility we have encouraged authors to use flow diagrams showing the logical connections between different chapters and to put some topics in starred sections. These cover more advanced and alternative material which is not required for the understanding of latter parts of each volume.

Although these books were conceived as a series, each of them is self-contained and can be used independently of the others. Several of them are suitable for wider use in other sciences. Each Author's Preface gives details about the level, prerequisites, etc., of that volume.

The Manchester Physics Series has been very successful with total sales of more than a quarter of a million copies.

We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, for helpful criticisms and stimulating comments. Our particular thanks go to the authors for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and often accepting, the suggestions of the editors.

Finally we would like to thank our publishers, John Wiley & Sons, Ltd, for their enthusiastic and continued commitment to the Manchester Physics Series.

D. J. Sandiford  
F. Mandl  
A. C. Phillips  
*February 1997*

# Author's preface

This book deals with the basic principles of nuclear physics and their many applications in the modern world. Much of the book is based on a course "Applications of Nuclear Physics", which has been offered as a one-semester, third-year option in physics at the University of Manchester for a number of years.

The book is in two parts. Part I is a brief general introduction to the principles of nuclear physics. However, the emphasis of the book is on applications. These form Part II, which presupposes only a knowledge of the basic concepts developed in Chapter 1 of Part I. The aim of Part II is to introduce the reader to a wide diversity of applications and the underlying physics rather than to attempt a complete coverage of the subject.

The book is addressed mainly to science and engineering students, who require knowledge of the fundamental principles of nuclear physics and its applications. Some of these students may wish later to take advanced courses in nuclear physics or specialized courses in different areas of nuclear science and technology such as nuclear chemistry, nuclear engineering, instrumentation, radiation biology and nuclear medicine.

The level of the book is suitable for undergraduates in their second and third years of physics education and a corresponding grounding in introductory physics and mathematics is assumed.

The approach to the different topics is mainly from an experimental point of view, with illustrative examples. Complex and extensive mathematical treatments generally are avoided. However, where possible, an attempt is made to give a proper understanding based on fundamental

physics principles. Derivations of formulae are given or outlined with a minimum of mathematical complexity. A bibliography contains references where much more extensive coverage can be found on all topics. Problem solving is an integral part of learning and understanding physics and, to that end, a set of problems is attached to each chapter. Hints and outlined solutions to all of them are collected together in an appendix.

The material in the book is designed to be used flexibly for a range of different courses. Part I could comprise a one-semester core course on the principles of nuclear physics, which introduces the main elements of nuclear structure, radioactivity and nuclear reactions. Part II can form the basis of different courses on applications allowing choice of topics and courses of different lengths. The flow diagram inside the front cover shows the logical connections between chapters and how material may be used selectively. In addition, much of the material in Part II can be tailored to meet specific needs. For example, if only the basic principles of a nuclear reactor are wanted, the sections in Chapter 10 dealing with the finite reactor, reactor operation and future uses may be omitted.

It is a pleasure to acknowledge the many friends and colleagues who have assisted me in the course of this work. These include Kevin Connell, John Hemingway, Tony Phillips, Roy Ryder, David Sandiford, Harbans Sharma and John Simpson. I am particularly grateful for the many thoughtful comments and helpful suggestions given to me by Bill Phillips and by Paddy Regan, who volunteered to read the entire manuscript.

Finally, I owe a special debt of gratitude to my editor Franz Mandl for his patient guidance throughout and for his critical comments and innumerable detailed suggestions, which contributed so much to the production and quality of the final manuscript.



February 2001  
**J. S. Lilley**

**PART I**

**PRINCIPLES**

# 1

## Introduction and Basic Concepts

### 1.1 INTRODUCTION

It was in 1896 that Becquerel in France detected, by chance, faint traces of the existence of the nucleus in the atom. For many years after that the study of nuclear physics remained a curiosity and intellectual challenge to scientists, but had little practical use outside its own field. The situation changed totally in the 1930s with discoveries that culminated in the cataclysmic demonstrations near the end of the second world war of the immense energy locked up by the force that holds the atomic nucleus together. An unprecedented and irrevocable step had been taken in the degree of power available to humankind with dramatic consequences for good and ill.

Today, nuclear physics has entered into our modern world in a significant way. It influences other branches of science: chemistry, biology, archaeology, geology, engineering, astrophysics and cosmology. It is used widely in society at large - in industry, the environment, medicine, defence, criminology, power production and many other areas. Applications are found even in religion and the arts, where equipment and methods developed originally for nuclear research have found novel application. However, the exploitation of such a powerful force carries with it some danger and is the subject of much debate.

The main aim of this book is to address the broad range and variety of the techniques and applications of nuclear physics used today. The basic physics underlying them is given first in order that the benefits and drawbacks can be properly appreciated. No particular stance is taken on controversial issues. The view taken is that a proper understanding of the subject is important and necessary in order that wise decisions can be taken about how nuclear energy and nuclear radiation should be used.

Essential nuclear physics for understanding the applications is given in this first chapter. Other chapters in Part I give further development of the topics introduced in Chapter 1. The coverage of the applications in Part II is by no means exhaustive. It is intended broadly to inform the reader and provide a suitable preparation for those students who plan to take more advanced courses on any of the separate topics.

Unlike atomic physics, which is underpinned by electromagnetism, there is no fundamental theoretical formalism that completely describes nuclei and nuclear behaviour. For example, there is no formula, analogous to Coulomb's law for the force between two electric charges, which exactly expresses the force between two basic constituents of the nucleus. Progress in understanding the nature of nuclei is made using approximate models, each of which provides insight into the complexity of the real situation, but with a limited range of applicability. Models are drawn from analogy with classical and other branches of physics and are formulated to be consistent with observed properties and behaviour. Conceptual models played a vital role in the first few decades of the twentieth century when the basic framework of the subject was being established. The following section is a short account of this early period.

## 1.2 EARLY DISCOVERIES

The history of the nucleus dates from the latter years of the nineteenth century with the observation by Becquerel in 1896 of the fogging of photographic plates by an unknown radiation emanating from uranium-bearing rocks. He had encountered *radioactivity*. Detailed studies of this new phenomenon began to be made, notably by Marie and Pierre Curie in France and by Ernest Rutherford, who had come to England from New Zealand earlier in 1895 to work in Cambridge with J. J. Thompson (who discovered the electron in 1897). It was soon revealed that there are three, distinctly different types of radiation emitted by radioactive substances. They were called alpha ( $\alpha$ ), beta ( $\beta$ ) and gamma ( $\gamma$ ) rays - terms which have been retained to this day.

The most far-reaching advances in the subject during this early phase were made by Rutherford. He and his co-workers, first in Canada (1898-1907) and later in Manchester, England (1907-1919), began an intensive study of the new radiations. All the laws governing radioactive decay were established. It was shown that  $\alpha$ - and  $\beta$ -radioactive decays change the nature of the element and that  $\alpha$  particles are helium nuclei. Beta particles were found to be the same as electrons, and  $\gamma$  rays were identified as energetic photons (electromagnetic radiation).

Rutherford used  $\alpha$  particles to probe the structure of the atom itself. It was already known that the atom consisted of positively charged and negatively charged components, but there were two very different models for describing how these components might combine to form an atom. The 'planetary' model assumed that light, negatively charged electrons orbit a heavy, positively charged nucleus. The problem with this model was that the electrons would be constantly accelerating and should radiate energy as

electromagnetic waves, causing the atom to collapse. In an alternative model, proposed by J. J. Thompson, the electrons are embedded and free to move in an extended region of positive charge filling the entire volume of the atom. Such an atom would not collapse, but Thompson had difficulty in developing his model. For example, he was never able to account successfully for the discrete wavelengths observed in the spectra of light emitted by excited atoms.

The crucial breakthrough came from experiments carried out by Rutherford and his team in Manchester, who were studying the passage of a particles through matter. It was noted that very thin foils of gold caused a particles to be deflected occasionally through large angles and even in the backward direction. Rutherford realized that this could not be due to the combined effect of a large number of small-angle deflections and could only be explained if the a particle had encountered a tiny, but heavy, charged entity less than 1/1000th of the atom in size. Undaunted by the fact that the planetary model should not exist according to classical theory, he proposed that the atom does consist of a small, heavy positively charged centre surrounded by orbiting electrons which occupy the vast bulk of the atom's volume. The simplest atom, hydrogen, consisted of a proton and a single orbital electron.

Many atomic masses were known to be approximately integer multiples of a basic unit of mass about 1% lighter than the mass of the hydrogen atom. For example, the atomic masses of carbon, nitrogen and oxygen, expressed in these units, are approximately 12.0, 14.0 and 16.0, respectively. However, there are notable exceptions, such as the element chlorine, which has an atomic mass of 35.5 of these units. The idea that an element could consist of differing *isotopes*, which are atoms whose nuclei have different masses but the same charge, was put forward by Soddy in 1911. This explained the existence of anomalous

atomic weights, like chlorine, but reinforced an idea, which was current at that time, of nuclei consisting of different numbers of protons and electrons bound together in some way. The proton-electron model persisted for many years until developments in quantum mechanics exposed its shortcomings. No one, for example, could explain why an electron with enough energy to be emitted in  $\beta$  decay was not emitted instantly. Indeed, an estimate of the energy required to keep an electron inside the nucleus (see Problem 1.3) was many times greater than the energies seen in  $\beta$  decay, and an attractive force strong enough to do this would have effects on atomic spectra, which were not observed.

Little progress was made until 1932, when James Chadwick proposed the existence of the *neutron*, an uncharged nuclear constituent whose existence had been anticipated by Rutherford as early as 1920.<sup>1</sup> Bothe and Becker, in 1930, had observed highly penetrating, uncharged radiation from the  $\alpha$ -particle bombardment of beryllium. In 1931, F. Joliot and I. Curie measured fast protons emerging from paraffin exposed to this radiation. They surmised that the protons were recoiling from being bombarded by electromagnetic radiation and deduced that, if this were the case, the photon energy would have to be 50MeV - more energetic than the estimated total energy released in the reaction. Chadwick compared the recoils of protons and nitrogen from different bombarded materials and correctly deduced the mass of the neutral radiation particle to be approximately equal to that of the proton. The neutron was the critical missing ingredient for understanding nuclei. Heisenberg, Majorana and Wigner then took steps to establish the framework that forms the basis of the modern picture of the nucleus consisting of *nucleons* (neutrons and protons) held together by a short-

range force whose strength is independent of the type of nucleon.

From the results of subsequent experiments, in which particles were collided at very high energies, it has become apparent that neutrons and protons themselves have an underlying structure composed of three sub-nucleonic particles, which have been called *quarks*. Particle physics deals with the world of the quark and all other particles still thought to be fundamental. These aspects are not dealt with in this text. The structure of neutrons and protons is discerned only at very high energies and, for all practical purposes concerning nuclear structure research and nuclear physics applications in the modern world, the neutron-proton model of the nucleus is entirely adequate.

## 1.3 BASIC FACTS AND DEFINITIONS

### 1.3.1 The nucleus and its constituents

An atomic nucleus is the small, heavy, central part of an atom consisting of  $A$  nucleons:  $Z$  protons and  $N$  neutrons;  $A$  is referred to as the mass number and  $Z$ , the atomic number. Nuclear size is measured in fermis (also called femtometres) where  $1\text{fm} = 10^{-15}\text{ m}$ .

The basic properties of the atomic constituents can be summarized as follows:

|          | charge | mass (u) | spin ( $\hbar$ ) | magnetic moment ( $\text{J T}^{-1}$ ) |
|----------|--------|----------|------------------|---------------------------------------|
| proton   | $e$    | 1.007276 | 1/2              | $1.411 \times 10^{-26}$               |
| neutron  | 0      | 1.008665 | 1/2              | $-9.66 \times 10^{-27}$               |
| electron | $-e$   | 0.000549 | 1/2              | $9.28 \times 10^{-24}$                |



**Charge:** Protons have a positive charge of magnitude  $e = 1.6022 \times 10^{-19} \text{C}$  (coulombs) equal and opposite to that of the electron. Neutrons are uncharged. Thus, a neutral atom ( $A, Z$ ) contains  $Z$  electrons and can be written symbolically as  ${}^A_Z X_N$ .

**Mass:** Nuclear and atomic masses are expressed in atomic mass units (u), based on the definition that the mass of a neutral atom of  ${}^{12}_6\text{C}_6$  is exactly 12.000 u ( $1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$  - see Appendix A).

**Spin:** Each of the atomic constituents has a spin  $1/2$  in units of  $\hbar$  ( $= h/2\pi$ ) and is an example of the class of particles of half-integer spin known as *fermions*. Fermions obey the *exclusion principle*, first enunciated by Wolfgang Paull in 1925 which determines the way electrons can occupy atomic energy states. The same rule applies to nucleons in nuclei, as we discuss in the next section.

**Magnetic moment:** Associated with the spin is a magnetic dipole moment. Compared with the magnetic moment of an electron, nuclear moments are very small. However, they play an important role in the theory of nuclear structure. It may be surprising that the uncharged neutron has a magnetic moment. This reflects the fact that it has an underlying quark sub-structure, consisting of charged components. An important application of nuclear moments, based on their behaviour in electromagnetic fields, is the technique of magnetic resonance imaging or nuclear magnetic resonance, which is described in Chapter 9.

## 1.3.2 Isotopes, Isotones and Isobars

Isotopes of an element are atoms whose nuclei have the same  $Z$  but different  $N$ . They have similar electron structure and, therefore, similar chemical properties. For example, hydrogen has three isotopes:  ${}^1_1\text{H}_0$ ,  ${}^2_1\text{H}_1$  and  ${}^3_1\text{H}_2$ , whose nuclei are, respectively, the proton  $p$ , the deuteron  $d$ , and the triton  $t$ . Carbon has three, naturally occurring isotopes:  ${}^{12}_6\text{C}_6$ ,  ${}^{13}_6\text{C}_7$  and  ${}^{14}_6\text{C}_8$ . Nuclei with the same  $N$  and different  $Z$  are called *isotones*, and nuclides with the same mass number  $A$  are known as *isobars*. In a symbolic representation of a nuclear species or *nuclide*, it is usual to omit the  $N$  and  $Z$  subscripts and include only the mass number as a superscript, since  $A = N + Z$  and the symbol representing the chemical element uniquely specifies  $Z$ .

### 1.3.3 Nuclear mass and energy

Inside a nucleus, neutrons and protons interact with each other and are bound within the nuclear volume under the competing influences of attractive nuclear and repulsive electromagnetic forces. This binding energy has a direct effect on the mass of an atom.

In 1905, Einstein presented the equivalence relationship between mass and energy:  $E = mc^2$ . The speed of light  $c$  is very large and so even a small mass is equivalent to a large amount of energy\*. The complete conversion of 1 g of matter releases about as much energy as 20000 tons of TNT. On the atomic scale, 1 u is equivalent to  $931.5 \text{ MeV}/c^2$ , which is why energy changes in atoms of a few electron volts cause insignificant changes in the mass of the atom. Nuclear energies, on the other hand, are millions of electron volts and their effects on atomic masses are easily detectable. For example, the mass of an atom can be measured to high precision (of the order 1 part in  $10^6$ ) in a

modern mass spectrometer, and the error in the mass-energy of  $^{12}\text{C}$ , measured to 1 part per million, is about 11 keV. Mass differences can be determined to even greater precision with these instruments, and many relative masses are known to an accuracy of a few keV.

Relative masses of nuclei can also be determined from the results of nuclear reactions or nuclear decay. For example, if a nucleus is radioactive and emits an  $\alpha$  particle, we know from energy conservation that its mass must be greater than that of the decay products by the amount of energy released in the decay. Therefore, if we measure the latter, we can determine either of the initial or final nuclear masses if one of them is unknown. An example of this is presented in Section 1.5.1.

The binding energy of a nucleus  $B$  is the energy required to separate it into its constituent neutrons and protons. The mass of an atom, therefore, is less than the mass of its constituents by the mass equivalent of  $B$ . Symbolically, this is written as

$$(1.1) \quad m(A, Z) = Zm_{\text{H}} + Nm_{\text{n}} - B/c^2$$

where  $m(A, Z)$ ,  $m_{\text{H}}$  and  $m_{\text{n}}$  are the atomic masses of the nuclide (mass number  $A$  and atomic number  $Z$ ), a hydrogen atom and a neutron, respectively, and  $B$  is expressed in energy units. This equation assumes that differences in the average binding energies of electrons in different atoms are negligible. All the effects of forces acting on the nucleons inside a nucleus are contained in the binding-energy term. Variations in atomic masses due to variations in binding energy are invariably small compared with an atomic mass unit, which is equivalent to nearly 1 GeV. For this reason, an atomic mass is often presented in the literature as the difference  $m(A, Z) - A$  between the mass (in atomic mass units) and the atomic number. This is called the *mass excess* ( $me$ ). Mass excesses of a number of nuclides are

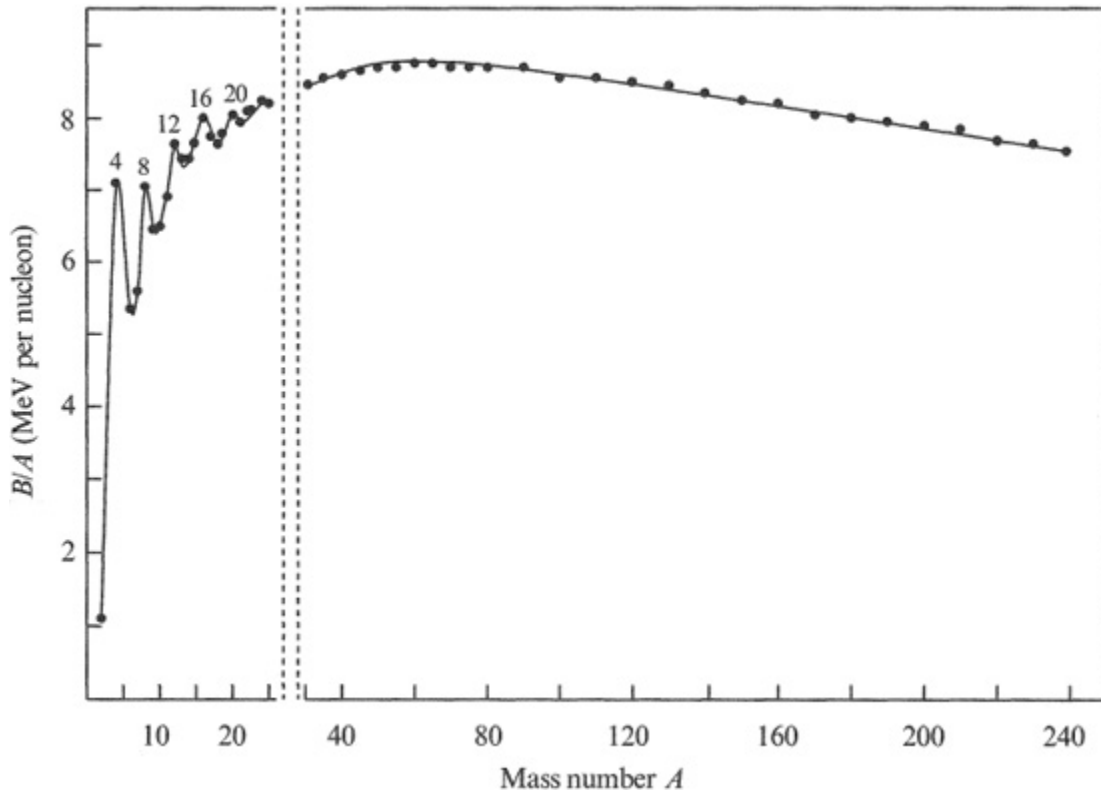
listed in Appendix F in units of micro atomic mass units ( $\mu u$ ).<sup>2</sup>

Nuclear binding energy increases with the total number of nucleons  $A$  and, therefore, it is common to quote the average binding energy per nucleon ( $B/A$ ). The variation of  $B/A$  with  $A$  is shown in [Figure 1.1](#) and reveals a number of important features. There is an initial sharp rise with  $A$  to a broad maximum of about 8.6 MeV per nucleon near a mass number of 60. This is followed by a gradual decrease to about 7.6 MeV per nucleon for the heaviest nuclei. Nuclei with  $A$  greater than 238 are not found in significant quantities in the earth's crust. Several sharp peaks in the light-nuclear region correspond to the nuclei  ${}^4\text{He}$ ,  ${}^8\text{Be}$ ,  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ ,  ${}^{20}\text{Ne}$  and  ${}^{24}\text{Mg}$ . The  ${}^4\text{He}$  nucleus ( $\alpha$  particle) has a particularly stable structure, and the  $A$  and  $Z$  of the other nuclei are multiples of the  $\alpha$  particle. Their extra stability is taken as evidence that they have a structure which bears some resemblance to that of a collection of a particles.

The overall form of the curve in [Figure 1.1](#) is the result of the combined effect of the nuclear and electrostatic (Coulomb) forces and is discussed in some detail in the next chapter. The nuclear force is very complex and it is not possible to deduce its form precisely. However, the important property we need for the present discussion is that it operates over a very short distance of a few fermis. In a light nucleus, where there are only a few nucleons, any one of them interacts with all the other nucleons and, therefore, its binding energy increases with mass number. However, the size of a nucleus increases with  $A$  and when it exceeds that of the range of the internucleon force, the nucleon will interact only with its neighbours which lie within that range. Consequently, the binding energy of a nucleon due to the nuclear force alone will tend to approach a constant value at large  $A$ . The Coulomb force, on the other

hand, acts over a much larger range, and the electrical potential energy per proton grows steadily as  $Z$  increases. It is of opposite sign to the nuclear binding energy and, eventually, this negative term becomes the determining factor causing the fall in  $B/A$  beyond  $A \sim 60$ .

[Figure 1.1](#) Binding energy per nucleon of stable nuclei plotted as a function of mass number  $A$ . The horizontal scale below  $A = 30$  is twice that above  $A = 30$ .



From the form of the binding energy curve, it is clear that when  $A \gtrsim 120$ , energy could be released by breaking the nucleus into two, roughly equal fragments. This is called *fission* and it occurs spontaneously if  $A$  is sufficiently large or if a very heavy nucleus, like  $^{235}\text{U}$  or  $^{238}\text{U}$ , is excited to a higher energy state. At the other end of the mass scale, energy can be gained by combining two light nuclei together in a *fusion* reaction to form a heavier system with higher  $B/A$ . The fission process is already being exploited for commercial power production, and nuclear fusion is a

potential source of energy for the future. Some of the underlying physics of fission and fusion is covered in Sections 2.2, 4.5 and 4.6. A fuller discussion of their practical importance is presented in Chapters 10 and 11.

## 1.4 NUCLEAR POTENTIAL AND ENERGY LEVELS

### 1.4.1 Nucleon states in a nucleus

It can be shown to a first approximation that a nucleon in a nucleus experiences an average (or mean) attractive energy due to the strong nuclear interaction with its neighbours. This is approximately constant in the nuclear interior and, for the uncharged neutron, it can be represented, as in [Figure 1.2\(a\)](#), by a potential well. This well does not have a sharp edge because of the range of the nuclear force and because the distribution of nucleons in the surface of a nucleus is diffuse. Outside the nucleus, the neutron experiences no force and its potential energy (PE) does not change until it approaches the nuclear surface. There, under the influence of the attractive force, it 'falls' into the nuclear interior with a gain in kinetic energy corresponding to the decrease in PE.

A proton experiences in addition a repulsive Coulomb PE,  $V_C$ , due to its positive charge. The general effect of this is shown in [Figure 1.2\(b\)](#). Outside the nucleus, where there is no nuclear force acting on the proton, the form of  $V_C$  is that due to a point charge. Inside the nucleus, the Coulomb energy reduces the depth of the total potential for protons compared with that for neutrons by an amount that