# Statistics rookit 

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## Introduction

This 'toolkit' is the second in our series and is aimed as a summary of the key concepts needed to get started with statistics in healthcare.


Often, people find statistical concepts hard to understand and apply. If this rings true with you, this book should allow you to start using such concepts with confidence for the first time. Once you have understood the principles in this book you should be at the point where you can understand and interpret statistics, and start to deploy them effectively in your own research projects.
The book is laid out in three main sections: the first deals with the basic nuts and bolts of describing, displaying and handling your data, considering which test to use and testing for statistical significance. The second section shows how statistics is used in a range of scientific papers. The final section contains the glossary, a key to the symbols used in statistics and a discussion of the software tools that can make your life using statistics easier.
Occasionally you will see the GO icon on the right. This means the difficult concept being discussed is beyond the scope of this textbook. If you need more information on this point you can either refer to the text cited or discuss the problem with a statistician.


Types of data: categorical, numerical, nominal, ordinal, etc. Describing your data (see p. 3)

For $\mathrm{H}_{0}$ and $\mathrm{H}_{1}$ (see p. 20)


Choosing the right type of test to use will prove the most difficult concept to grasp. The book is set out with numerous examples of which test to use; for a quick reference refer to the flow chart on page 26

Once you have chosen the test, compute the value of the statistic and then leam to compare the value of this statistic with the critical value (see p. 20)

Start with Excel (learn how to produce simple graphs) and then move on to SPSS and when your skills are improving consider STATA, SAS or R (see p. 103)

## Data: describing and displaying

The type of data we collect determines the methods we use. When we conduct research, data usually comes in two forms:

- Categorical data, which give us percentages or proportions (e.g. ‘60\% of patients suffered a relapse').
- Numerical data, which give us averages or means (e.g. 'the average age of participants was 57 years').
So, the type of data we record influences what we can say, and how we work it out. This section looks at the different types of data collected and what they mean.


## Any measurable factor, characteristic or attribute is a variable

A variable from our data can be two types: categorical or numerical.
Categorical: the variables studied are grouped into categories based on qualitative traits of the data. Thus the data are labelled or sorted into categories.


A special kind of categorical variables are binary or dichotomous variables: a variable with only two possible values (zero and one) or categories (yes or no, present or absent, etc.; e.g. death, occurrence of myocardial infarction, whether or not symptoms have improved).
Numerical: the variables studied take some numerical value based on quantitative traits of the data. Thus the data are sets of numbers.


You can consider discrete as basically counts and continuous as measurements of your data.

Censored data - sometimes we come across data that can only be measured for certain values: for instance, troponin levels in myocardial infarction may only be detected for a certain level and below a fixed upper limit (0.2-180 $\mu \mathrm{g} / \mathrm{L}$ )

## Summarizing your data

It's impossible to look at all the raw data and instantly understand it. If you're going to interpret what your data are telling you, and communicate it to others, you will need to summarize your data in a meaningful way. Typical mathematical summaries include percentages, risks and the mean.
The benefit of mathematical summaries is that they can convey information with just a few numbers; these summaries are known as descriptive statistics.
Summaries that capture the average are known as measures of central tendency, whereas summaries that indicate the spread of the data usually around the average are known as measures of dispersion.

## The arithmetic mean (numeric data)

The arithmetic mean is the sum of the data divided by the number of measurements. It is the most common measure of central tendency and represents the average value in a sample.
$\bar{x}=\frac{\Sigma x_{1}}{n}$
Consider the following test scores:

| Test scores out of ten |  |  |  |
| :---: | :--- | ---: | :---: |
|  | 6 | 4 |  |
| 4 | 7 |  |  |
| 5 | 2 |  |  |
| 6 | 9 |  |  |
| 7 | 7 | 1 |  |

> | $\bar{x}=$ sample mean |
| :--- |
| $\mu=$ population mean |
| $\Sigma=$ the sum of |
| $x=$ variable |
| $i=$ the total variables |
| $n=$ number of measurements |

2. Divided by the number of measurements
$(6+4+5+6+7+4+7+2+9+7) / 10=5.7$
3. The sum of the measurements 3. Gives you the mean

To calculate the mean, add up all the measurements in a group and then divide by the total number of measurements.

## The geometric mean

If the data we have sampled are skewed to the right (see p. 7) then we transform the data using a natural logarithm (base e $=2.72$ ) of each value in the sample. The arithmetic mean of these transformed values provides a more stable measure of location because the influence of extreme values is smaller. To obtain the average in the same units as the original data - called the geometric mean - we need to back transform the arithmetic mean of the transformed data:

## The weighted mean

The weighted mean is used when certain values are more important than others: they supply more information. If all weights are equal then the weighted mean is the same as the arithmetic mean (see p. 54 for more).

We attach a weight ( $w_{i}$ ) to each of our observations ( $x_{i}$ ):

$$
\frac{w_{1} x_{1}+w_{2} x_{2}+\ldots w_{2} x_{n}}{w_{1}+w_{2}+\ldots w_{n}}=\frac{\sum w_{1} x_{1}}{\sum w_{1}}
$$

## The median and mode

The easiest way to find the median and the mode is to sort each score in order, from the smallest to the largest:

| 1) 2 | 6) 6 | In a set of ten scores take the fifth and sixth values |
| :---: | :---: | :---: |
| 2) 4 | 7) 7 | $\\|$ |
| 3) 4 | 8) 7 | $(6+6) / 2=6$ |
| 4) 5 | 9) 7 | $\pi$ |
| 5) 6 | 10) 9 | The median is equal to the mean of the two middle values or |

The median is the value at the midpoint, such that half the values are smaller than the median and half are greater than the median. The mode is the value that appears most frequently in the group. For these test scores the mode is 7 . If all values occur with the same frequency then there is no mode. If more than one value occurs with the highest frequency then each of these values is the mode. Data with two modes are known as bimodal.
Choosing which one to use: (arithmetic) mean, median or mode?
The following graph shows the mean, median and mode of the test scores. The $x$-axis shows the scores out of ten. The height of each bar ( $y$-axis) shows the number of participants who achieved that score.

