Intelligent Systems, Control and Automation: Science and Engineering

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Xinwei Wang · Jie Liu · Haijun Peng

Symplectic Pseudospectral Methods for Optimal Control

Theory and Applications in Path Planning

Springer

Intelligent Systems, Control and Automation: Science and Engineering

Volume 97

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Xinwei Wang
Department of Engineering Mechanics
Dalian University of Technology
Dalian, Liaoning, China

Jie Liu
War Research Institute
Academy of Military Sciences
Beijing, China

Haijun Peng
Department of Engineering Mechanics
Dalian University of Technology
Dalian, Liaoning, China

ISSN 2213-8986 ISSN 2213-8994 (electronic)
Intelligent Systems, Control and Automation: Science and Engineering
ISBN 978-981-15-3437-9 ISBN 978-981-15-3438-6 (eBook)
https://doi.org/10.1007/978-981-15-3438-6

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Preface

Vehicles are equipment with capabilities to realize certain missions in various workspace such as land, sea, air, and space. Their structures usually exhibit complex dynamical characteristics and dynamical behaviors. Mechanical disciplines such as analytical dynamics, multi-body dynamics, and nonlinear dynamics give deep insights into motion laws of vehicles. Meanwhile, control disciplines provide feasible and powerful tools to regulate the motion of vehicles according to human will.

In the 1960s, Richard Bellman (academician of United States National Academy of Sciences) and Lev S. Pontryagin (academician of Academy of Sciences of former Soviet Union) had laid the mathematical foundation of optimal control theory. Optimal control theory greatly facilitated the progress of military science and equipment during the cold war, successively providing cutting edge technologies for the two countries for tens of years. Nowadays, the optimal control problems for vehicles encountered in engineering are becoming more and more complicated, which makes it impossible to implement analytical solutions by Bellman's dynamic programming and Pontryagin's maximum principle. Arthur E. Bryson (the academician of both United States National Academy of Sciences and American Academy of Arts and Sciences), who is considered as the father of modern optimal control theory, points out that "with powerful digital computer, numerical solutions can be found for realistic problems". It significantly prompts the development of computational optimal control.

The trajectory optimization of vehicles is a typical open-loop optimal control problem for nonlinear systems. It aims at finding optimal control inputs and corresponding trajectory to fulfill the mission, meanwhile minimizing a certain objective and satisfying various kinds of constraints. Direct methods and indirect methods are the two main categories of computational optimal control techniques. They both usually start from the difference discretization of dynamic equations while neglecting the inherent dynamical characteristics of optimal control problems. This leads to deficiencies in the stability and precision of computational optimal control techniques. Jerrold E. Marsden, the fellow of British Royal Society and the leading scholar in classical mechanics, says that "Algorithms could be developed with the natural dynamics built in, thereby yielding better convergence properties". In addition, Gene H. Golub, the founder of modern matrix computation, points out that "It is a basic tenet of numerical analysis that solution procedures should exploit structure whenever it is present." For optimal control problems, they have inherent Hamiltonian mathematical structures. Thus, the energy/momentum variation of the controlled system with respect to time can be seen as important *natural dynamics*, and the symplectic structure is a distinct *mathematical structure* in Hamiltonian systems.

State-space representation is the basis of optimal control theory and it can trace back to the system of Hamiltonian canonical equations. Hamiltonian systems, as the cornerstone in analytical dynamics, bridge the gap between optimal control and analytical mechanics. Based on Hamiltonian systems, Wanxie Zhong (the academician of Chinese Academy of Sciences) identified the simulation between structural mechanics and optimal control for the first time. And a series of computational optimal/robust control methods for linear systems have been proposed drawing ideas in computational mechanics.

The first and the third authors of the book, i.e., Xinwei Wang and Haijun Peng, were both taking Ph.D.s under the supervision of Prof. Zhong. In the last two decades, based on Hamiltonian systems and the symplectic theory, the research group has developed a series of symplectic algorithms to solve nonlinear optimal control problems with different complex features. In this book, we report our recent progress in symplectic numerical algorithms that incorporate pseudospectral methods to achieve better computational efficiency and accuracy. Additionally, Jie Liu, the second author of the book, has applied the developed symplectic pseudospectral method to solve trajectory optimization problems in various fields. Hence, some of his works are taken as examples in this book.

The publication of the book should appreciate those who helped us over the years. Our first gratitude goes to Prof. Zhong who was the pathfinder in our research direction. And many thanks go to other colleagues from the Dalian University of Technology, including Prof. Qiang Gao, Prof. Zhigang Wu, Prof. Shujun Tan, etc. Gratitude also goes to Prof. Wei Han from Naval Aviation University. Besides, many of our current and former students, such as Mingwu Li, Xin Jiang, Boyang Shi, also contributes much to the development of algorithms during the last decade. Finally, we gratefully acknowledge helpful suggestions and assistance by Jasmine Dou (Editor, Springer) and her staff.

Dalian, China — Xinwei Wang
Beijing, China — Jie Liu
Dalian, China — Haijun Peng
July 2020

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Chapter 1
Overview of This Book

1.1 Optimal Control

Optimal control is an important component of modern control theory [1, 2]. In the 1960s, the former Soviet Union scholar Lev Pontryagin proposed the Pontryagin's maximum principle [3], meanwhile, the American scholar Richard Bellman developed the dynamic programming method [4]. Such two techniques significantly enriched and improved the theory of optimal control, prompting the development of the analytical solution of optimal control problems. Humans were initially starting the exploration of the space at that time, techniques based on optimal control theory such as orbit design [5] and the Kalman filter [6] won great successes.

However, as optimal control problems encountered in engineering are getting more complicated (i.e., large-scale, complex constraints, time-delay, etc.), the analytical solution becomes impossible. Luckily, due to the rapid development of computational techniques and devices, it is available for engineers to implement numerical solutions. During the last 70 years, the optimal control theory has been widely applied to various fields such as aerospace engineering, chemical engineering, economics, communications, automobile engineering. Hence, computational optimal control techniques [7–9], as the core to implement optimal control theory in practice, have drawn more and more attention.

1.2 Pseudospectral Methods

Numerical methods for optimal control problems are generally divided into indirect methods and direct methods. Pseudospectral methods, as the most popular direct methods in the last two decades, have drawn much attention [10]. The collocation

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https://doi.org/10.1007/978-981-15-3438-6_1

points in pseudospectral methods are generally orthogonal Gauss points, leading to a highly accurate approximation of the original problem. More precise solutions can be obtained by pseudospectral methods when compared to other direct schemes under the same scale of discretization. Additionally, pseudospectral methods have a faster convergent rate, i.e., they show exponential convergent rate for problems where solutions are smooth and well behaved. Though this excellent property loses for constrained problems, multi-interval strategies can be used for compensation, leading to local pseudospectral methods. The first successful practical application of pseudospectral methods is the zero propulsion attitude maneuver of Internal Space Station in 2007. The operation command generated by pseudospectral methods helps to save fuel that values about one million dollars [11].

1.3 The Property of Symplectic Conservation

Optimal control problems can be transformed into Hamiltonian systems by the Pontryagin's maximum principle or the variational principle. The most notable feature of Hamiltonian systems is the phase flow in Hamiltonian systems is a symplectic transformation. Numerical methods that are symplectic conservative can solve Hamiltonian systems efficiently [12]. Hence, computational optimal control methods owning the symplectic conservative property are much appealing.

The concept of symplectic conservation is first proposed in computational mechanics. Hence, many scholars draw on mature theories in computational mechanics to enrich the symplectic conservative application in computational optimal control. For example, Zhong et al., explored the simulation between computational structure mechanics and optimal control [13]. Marsden extended the variational integration methods in computational dynamics to computational optimal control and proposed the Discrete Mechanics and Optimal Control (DMOC) method based on the Lagrange-D'Alembert's principle in analytical mechanics [14, 15]. Recently, Peng et al. developed a series of symplectic methods based on the generating function method [16–19].

Computational techniques that own the property of symplectic conservation has three most notable merits. First, the Hamiltonian structure of the original system is conserved; Secondly, they have good stability for problems with a long time interval; Thirdly, they can precisely reflect the energy variation for mechanical systems. Hence, it is attractive to construct symplectic numerical methods for optimal control problems.

1.4 Motivation of the Book

For decades, pseudospectral methods are generally developed and improved under the framework of direct methods. However, as essentially a numerical approximation technique, the application of pseudospectral methods should not be limited to the construction of direct methods. Additionally, during the design of traditional numerical methods, one usually focuses on how to improve the numerical precision under given discretization scheme while the inherent mathematical structures of optimal control problems are neglected. Based on the above facts, this book will start from the inherent Hamiltonian mathematical structure of optimal control problems and develop a series of symplectic pseudospectral methods (SPMs) for problems with different features. The parametric variational principle and the multi-interval pseudospectral methods are used when constructing SPMs. Additionally, the SPM is taken as the core solver to construct symplectic pseudospectral model predictive controllers. Finally, some successful applications of SPMs in solving path planning problems are presented.

1.5 Scope of the Book

The book is constituted of three parts. Part I (this chapter, Chaps. 2, and 3) is the introductory material. Part II (Chaps. 4–7) provides a series of symplectic pseudospectral methods for nonlinear optimal control problems with various complicated factors. Part III (Chaps. 8–11) gives the application of symplectic pseudospectral methods in trajectory planning of various vehicles. The detailed content of each chapter is summarized as follows:

Part I: Introductory materials

This chapter gives an overview of this book, where the motivation of this book is emphasized.

Chapter 2 summarizes the numerical methods for nonlinear optimal control problems. Four kinds of computational techniques, i.e., indirect methods, direct methods, hybrid methods, and artificial intelligence-based methods are reviewed. And a brief comparison between indirect methods and direct methods is provided.

Chapter 3 first gives the mathematical formulations of three kinds of problems studied in this book. And mathematical foundations required when constructing SPMs, such as the Hamiltonian structure of optimal control problems, symplectic theory and pseudospectral methods are briefly presented.

Part II: Symplectic pseudospectral methods for nonlinear optimal control problems

Chapter 4 focuses on generally unconstrained nonlinear optimal control problems. An SPM for this kind of problem is developed based on the first kind of generating function. Besides, a mesh refinement technique based on the relative curvature of state variables is provided.

Chapter 5 focuses on problems with inequality constraints.

Chapter 6 focuses on time-delayed problems.

Chapter 7 presents the basic idea of model predictive control. And the SPM developed in Chap. 5 is taken as the core solver to construct the symplectic pseudospectral model predictive controller.

Part III: Applications in trajectory planning and control

Chapter 8 provides several applications in the optimal maneuver of the spacecraft.

Chapter 9 presents trajectory planning problems for unmanned ground systems with different configurations.

Chapter 10 presents an autonomous control framework of three-dimensional overhead cranes.

Chapter 11 focuses on trajectory planning issues for tractor-trailer systems.

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Chapter 2
Computational Techniques for Nonlinear Optimal Control

2.1 Introduction

There are already many excellent reviews on numerical techniques for nonlinear optimal control [1–5]. According to the implementation of numerical techniques, computational methods for optimal control problems can generally fall into two groups, i.e., direct methods and indirect methods. Besides, hybrid methods and artificial intelligence-based methods are also popular. In this chapter, the pros and cons of these methods are presented and researches that consider the property of symplectic conservation are reviewed.

2.2 Indirect Methods

Indirect methods transform an optimal control problem into a Hamiltonian two-point boundary value problem (TPBVP) of state and costate variables by the variational principle or the Pontryagin's maximum principle. The TPBVP may couple with other algebraic equations if complicated factors are involved in the optimal control problem, and it is called the first-order necessary conditions [6]. Then the numerical solver for TPBVPs is used to solve the problem.

The main advantage of indirect methods is twofold. On the one hand, numerical solutions obtained by the indirect methods are naturally local optimal since it is constructed based on the first-order necessary conditions. On the other hand, the solutions of costate variables are usually directly provided, which are the prerequisite to analyze the Hamiltonian structure of the original optimal control problem. As for the disadvantage of indirect methods, the most notable one is that one must derive the first-order necessary conditions of the problem according to its formulation,

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https://doi.org/10.1007/978-981-15-3438-6_2

while this process would be extremely hard once complicated factors are involved. Fortunately, many mathematicians have done much fundamental work in this area, facilitating successive researchers to conduct deeper research under the framework of indirect methods. In [6], the first-order necessary conditions for problems with factors such as equality constraints, inequality constraints, interior point constraints, and integral constraints are provided. [7] focus on time-delayed optimal control problems. The first-order necessary conditions for fractional order optimal control problems are studied in [8, 9]. Another drawback of indirect methods is that high-quality initial guesses of state and costate variables are usually required. However, it is generally hard to provide a high-quality initial guess of costate variables since it is of no physical meaning.

For the transformed TPBVP, numerical methods such as shooting method [10–12], multiple shooting method [13], generating function method [14–17], and finite difference method [18–20], can be used. In shooting methods, initial guesses on costate variables must satisfy the transversality conditions. For problems with a short time interval, the shooting method is effective; however, for problems with a long time interval, it is prone to fail the convergence due to the numerical illness. It is seen that shooting methods are with small convergent radius and extremely sensitive to the initial guesses. Compared to shooting methods, the demand for initial guesses is not that high in the generating function method. However, plenty of series expansions are required in the generating method, one must turn to mathematical software such as MATLAB for help.

It should be noted that the Hamiltonian TPBVP structure naturally exists under the framework of indirect methods, which tends to facilitate the construction of symplectic methods. To realize the symplectic solving of linear-quadratic optimal control problems, Zhuk et al. design the symplectic Möbius integrator to solve Riccati equations in [21]. The generating function method mentioned above is also symplectic. Besides the initial development of Park et al., Pent et al. conduct some creative work in this area in recent years [22–25]. Focusing on the unconstrained nonlinear optimal control problem, they use the multiple interval mesh with regular Lagrange interpolation to transform the original problem into a system of nonlinear algebraic equations. Since the methods are constructed based on the least action principle and the finite element discretization, the core matrix is naturally sparse and symmetric, which is an advantage for solving large-scale problems. Later, Li et al. approximate state and costate variables by pseudospectral methods instead of the regular Lagrange interpolation [26]. Results suggest that discretization with pseudospectral methods lead to better computational efficiency and precision. In addition, focusing on problems with inequality constraints, Li et al. first transform the original nonlinear problem into a series of linear-quadratic problems by the quasilinearization technique. Then the transformed linear-quadratic problems are solved by symplectic algorithms in an iterative manner [27, 28]. The quasilinearization technique used in [27, 28] is actually a successive convexification technique. Due to its benefit, the initial guesses on costate variables are avoided, and the method is less sensitive to initial guesses.

2.3 Direct Methods

Direct methods transform a nonlinear optimal control problem into a finite-dimensional nonlinear programming (NLP) by discretization or parametrization [29]. Thus, NLP solvers can be used to solve the problem efficiently. The most notable advantage of direct methods is that they can treat the problem under a uniform NLP framework regardless of the characteristics of the problem. Optimal control problems met in engineering applications usually are subjected to complicated factors. Direct methods are popular in engineering due to this uniformity. In addition, direct methods usually have a larger convergent radius than indirect methods, which suggests they are less sensitive to initial guesses.

The disadvantages of direct methods mainly come down to three aspects. First, the scale of the resulted NLP grows faster than the number of variables, leading to the so-called "curse of dimensionality" [29]. Though computational devices nowadays have more and more computational power, this is still a bottleneck for large-scale problems. Secondly, constraints are usually slightly relaxed in direct method for numerical computation. Hence, original constraints may not be strictly satisfied in the obtained results. Thirdly, solutions obtained by direct methods usually do not satisfy the first-order necessary conditions for optimal control problems, which suggests that it even cannot guarantee the local optimality of the solutions. Information on costate variables is usually not directly available in direct methods, making it hard to analyze the Hamiltonian structure of the optimal control problem.

There are mainly two manners to implement the transformation to NLP by parametrization, i.e., function expansion [30, 31] and function interpolation. And the functional interpolation is commonly used. In methods based on function interpolation, control and/or state variables are approximated by a set of trial functions. The system equations and constraints are satisfied at a set of collation nodes within the time domain, and the cost functional is transformed as a function of state and control variables at the interpolation nodes. If the solution domain is divided into sub-intervals where state and controls are interpolated within the sub-interval, it leads to the local interpolation method. Conversely, if the solution domain is seen as a whole interval and the variables are interpolated within this big interval, it leads to the global interpolation method.

There are also some researches considering the symplectic conservation under the framework of direct methods. In [32], Bonnans discusses the symplectic conservation condition of using the Runge–Kutta difference scheme to discretize unconstrained nonlinear optimal control problems. The discrete mechanics and optimal control method (DMOC) mentioned in Sect. 1.3, is also a symplectic method, where the variational principle in discrete fashion is applied to achieve symplectic conservation [33, 34]. However, it should be noted that only symplectic conservation in system equations is considered. Hence, we say that the symplectic-conservation property in the DOMC method is not complete.

The book aims at developing symplectic methods that integrate the pseudospectral approximation. Hence, we will give a detailed introduction of pseudospectral methods for solving nonlinear optimal control problems in the rest of this section.