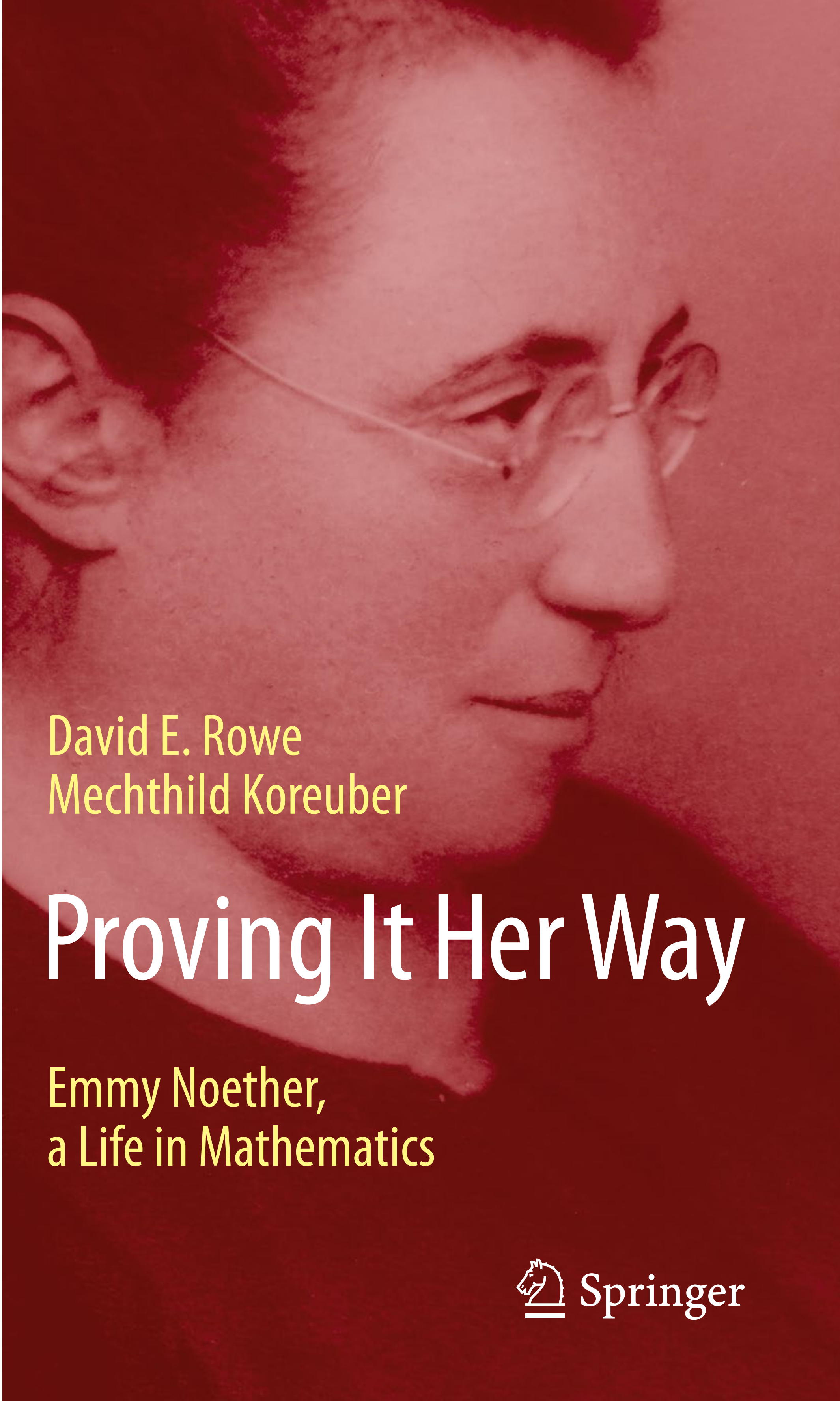

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Proving It Her Way

Emmy Noether, a Life in Mathematics

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Preface

Producing a book normally involves a long, arduous process that some authors feel compelled to describe in its preface. We have no such inclination, but even if that were the case, the truth is that this book came about as an afterthought. It arose out of our longstanding fascination with the career of Emmy Noether, but also from recent circumstances and events that made this project too tempting to resist. In 2019, the second author organized a major conference at the Freie Universität Berlin in cooperation with the Berlin Mathematics Research Center MATH+ and the Max Planck Institute for the History of Science to commemorate the hundredth anniversary of Noether's Habilitation in Göttingen.[1] This event, which took place on June 4, 1919, was not only a major milestone for Noether herself but also for women longing to pursue academic careers in Germany.

As one of the highlights of the conference, the ensemble *portraittheater Vienna* presented the premiere performance of their play, "Mathematische Spaziergänge mit Emmy Noether" [Schüddekopf/Zieher 2019]. Both of us, as historians of mathematics, had been involved in its production, and we were delighted by the result. So the idea of adapting the script for an English-speaking audience occurred to us right away. It must be said, though, that neither of us had any idea how Sandra Schüddekopf and Anita Zieher would manage to stage a play about a mathematician, one whose work even her peers found to be highly abstract. Nevertheless, they found a very elegant way to finesse that problem, and in a manner that would have appealed to Emmy Noether, whose personality shines through despite the handicap that most of her audience has absolutely no idea what she's really talking about.

It is worth mentioning that most of those present at the opening that evening were *not* mathematicians, because this and subsequent performances have shown that nearly anyone can enjoy this play. It was written, in fact, with the idea of introducing Emmy Noether to audiences that lack all the necessary prerequisites for truly understanding her, save one: curiosity. Viewers need not know anything about higher mathematics in order to grasp why Noether was a uniquely dynamic figure, a woman whose life story deserves to be known beyond the small sphere of those who happen to have heard about commutative rings. She loved to walk and talk and swim, while all the time living out her great passion for mathematics. Most who knew her from Göttingen, where she became a fixture of its famous mathematical community, thought of Emmy Noether as a simple woman who thoroughly enjoyed the simple things in her life, whether doing mathematics or eating pudding with her students and friends.[2] This more visible, light-hearted demeanor was, however, only one side of Emmy Noether's personality, and she

[1] The interdisciplinary character of the conference – which brought together mathematicians, physicists, historians of science, gender researchers, and cultural historians – is reflected in the forthcoming conference volume [Koreuber 2021].

[2] The latter activity, a major motif in the play, is described in [Dick 1970/1981, 1981: 49]: "The pudding always tasted the same – delicious."

had to endure many sorrows in her life. Barely had she arrived in Göttingen in the spring of 1915, when she received the news of her mother's death. This was only the first of four trips she would make back to Erlangen to mourn the passing of a family member: her father and also two of her three younger brothers. On such occasions, her calm dignity greatly impressed those who witnessed how she carried herself. Emmy Noether was by no means a one-dimensional type who lived for mathematics and nothing else, and Anita Zieher (Fig. 1.1) truly brings her full personality back to life on stage, now in the new adaptation of the original play, "Diving into Math with Emmy Noether" [Schüddekopf/Zieher 2020].[3]

Our book is thus, in the first instance, a companion to this version of the play. In both cases, the aim is to provide a window on the life of an extraordinarily creative mathematician, taking due account that her mathematical world is only truly accessible to those familiar with higher mathematics. Naturally, we have made many allusions to that world which will resonate for readers acquainted with modern algebra and abstract mathematical conceptions, but we have refrained from offering any details or explanations. So the present book remains, so to speak, on the surface by offering only vague hints of Emmy Noether's "real world," a realm open only to those who have "satisfied the prerequisites." For that audience – those seeking a more comprehensive picture of Noether's work and its central place in the mathematical world of her time – we recommend the volume *Emmy Noether – Mathematician Extraordinaire* [Rowe 2020b], written for readers who, let us say, have a vague recollection of Noetherian rings. Even though these two books are intended for quite different audiences, the goal in both cases is the same: to illuminate Noether's life in the context of her times by conveying a full-blooded picture of her role in shaping the mathematical activity of her day and, as it happened, well beyond.

For many people – including some with otherwise broad cultural interests – the very idea that doing mathematics might be a truly creative intellectual activity sounds a bit strange, if not wholly absurd. Yet for Emil Artin, one of Emmy Noether's close friends and to some extent a kindred spirit, mathematics was an art, not a science.[4] This, to be sure, has long been a contested position; indeed, it was a favorite topic of friendly disputes between Emmy and her brother Fritz, a leading applied mathematician who worked on problems like modeling turbulence in dynamical systems. Their father, Max Noether, was a different type of mathematician still, as will be seen in Chapter 3, when we turn to their years together in Erlangen. Mathematical talent ran through the Noether family, which led the Göttingen number-theorist Edmund Landau to liken Emmy Noether's kinfolk with a coordinate system in which she occupied the origin [Dick 1970/1981, 1981: 95].

In a book like this one, it would be pointless to attempt anything like a theoretical discussion of various types of mathematical creativity. Instead, we

[3] For information or to book a performance, contact office@portraittheater.net.

[4] On the broader context of earlier debates over the status of mathematics as art or science, see [Rowe 2018a, 401–411].

simply want to claim that the label mathematician, when applied to elites like the Noethers, fails to convey the vast range of intellectual activity covered by the term. In the case of Emmy Noether, this is a crucially important point; her way of thinking about mathematics using abstract concepts, rather than concrete objects, was by no means new. She became, however, the foremost exponent of this approach to mathematical theorizing, which she promoted in a radical manner, a style quite unlike that of any other contemporary figure. She and Artin also both believed that all truly deep mathematical truths must be beautiful. One can only begin to understand what that means, of course, by delving deeply into the mathematical world they shared, as many famous figures who came after them did. We, on the other hand, will be satisfied if this striking and, surely for many people, surprising aspect – mathematical insight as a special type of aesthetic experience – is at least dimly reflected in our account of Noether's visionary work. Or, to put this somewhat more ambitiously, we hope to show that people like her, who engage in mathematical research at the highest levels, can and should be considered as artists of a special kind. Still, we can only illumine such creative work indirectly, through the framework of its more mundane manifestations.[5] We nevertheless hope that in this way the mysterious archipelago of mathematical theories and ideas – some dubbed "abstract nonsense" by mathematicians themselves – will appear more transparent, less strange, and above all more clearly rooted in the larger realm of human intellectual endeavors.[6]

Parallels between our book and the play abound throughout, so we begin by highlighting information directly relevant to what unfolds onstage. Thus, Chapter 1 offers some skeletal facts concerning Emmy Noether's life along with brief information about the four other mathematicians who appear in cameo roles. Quite a few other people are mentioned on stage as well – most of them familiar figures in the world of mathematics – so this chapter closes with a glossary identifying several others whose lives intersected with Noether's. All of them surface in various places in the chapters that follow. Chapter 2 then moves beyond the bare bones of Emmy Noether's life by providing a rounded portrait without undue attention to details. With these first two chapters in place as background information, we invite readers to dive down deeper in those that then follow. Although arranged chronologically, they also focus on important themes and historical events that shaped different phases of Noether's life.

Emmy Noether is justly famous as the "mother of modern algebra," but it is important to understand what she meant by "doing algebra." Her vision of its role in mathematics did not seek to erect clear disciplinary boundaries setting algebraic investigations off from those in other fields. On the contrary, her work was closely tied to a larger trend that aimed to algebraicize other fields, from complex functions and number theory to topology, i.e. major parts of all mathematical

[5] In this respect, we follow the lead of Carol Parikh, author of *The Unreal Life of Oscar Zariski* [Parikh 1991], his real life being, of course, his mathematical works.

[6] The same motivation is central to the earlier study [Koreuber 2015], which approaches Emmy Noether's unique influence from the perspective of philosophy of science.

knowledge. In this respect, she clearly identified with words she once cited from Leopold Kronecker's 1861 inaugural address, when he was inducted into the Berlin Academy: "algebra is not actually a discipline in itself but rather the foundation and tool of all mathematics."[7] Not that many of Noether's contemporaries shared this view; far from it. Nor should we imagine that Noether meant this literally; she was well aware of the vast fields in analysis and applied mathematics that lie well beyond the realm of even her all-embracing view of algebraic research. Still, she was clearly the leading spokesperson of her generation for a position that many of her contemporaries found extreme.

One of Noether's closest collaborators, Helmut Hasse, clearly recognized the import of Noether's message, but he also sensed the need to spread the word. In a lecture on "The modern algebraic Method," he said:

> The aim of my talk is to promote modern algebra among non-specialists instead of preaching to the choir. It is not my intention to lure anyone from his field of specialization to become an algebraist. I see my task, rather, as laying the groundwork for a favorable understanding of modern algebra, helping to establish its methods – insofar as they are of general importance, and integrating these methods into the common knowledge of contemporary mathematicians. [Hasse 1930, 22]

Noether's former Göttingen colleague, Hermann Weyl, on the other hand, had deep misgivings, though not so much with regard to abstract algebra per se. What concerned Weyl was the general trend toward abstraction in mathematical research. In his famous memorial address [Weyl 1935], delivered at Bryn Mawr College on April 26, 1935, twelve days after Noether's sudden death, Weyl alluded directly to their differences with regard to the fertility of abstraction. A number of other things Weyl said on that occasion have often been discussed in the by now quite vast literature on Emmy Noether, though many of his most insightful remarks have been overlooked or forgotten. Several commentators have derided Weyl for his condescending remarks about her lack of sophistication and femininity. Her friend Pavel Alexandrov repeated his remark that "no one could contend that the Graces had stood by her cradle," nodding in assent before he immediately went on to underscore what really mattered. Fräulein Noether, as all who knew her well could attest, "loved people, science, life with all the warmth, all the joy, all the selflessness and all the tenderness of which a deeply feeling heart – and a woman's heart – was capable" [Alexandroff 1935, 11].

Alexandrov's tribute to Emmy Noether took place in Moscow on a very different occasion, some five months after her death. He spoke (in Russian) as President of the Moscow Mathematical Society on 5 September 1935, during a week-long international conference on topology. One of the mathematicians in the audience, as acknowledged by the speaker, was Fritz Noether, Emmy's brother.

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[7] [Noether 1932c]; the relevance of this citation is discussed in [Koreuber 2015, 225] as well as in [Merzbach 1983, 161].

Soon after she arrived at Bryn Mawr, Emmy Noether had undertaken efforts to find a position for him in the United States, but without success (Section 9.3). He and his two sons, Hermann and Gottfried Noether, then emigrated from Breslau to the Soviet Union, where Fritz took up work at the Mathematical Institute in Tomsk.

When Alexandrov spoke, he began by noting Weyl's address as well as B.L. van der Waerden's obituary article [van der Waerden 1935] in order to say that his remarks would have a somewhat different character. "I would like to evoke for you as accurate an image as possible of the deceased, as a mathematician, as the head of a large scientific school, as a brilliant, original, fascinating personality" [Alexandroff 1935, 2]. Alexandrov spoke movingly about the woman he and his friend Pavel Urysohn first met in Göttingen in 1923. Her school then had only just begun and consisted of a mere handful of German students, but its international character would soon thereafter unfold. A major breakthrough came the very next year with the arrival of B.L. van der Waerden from Amsterdam. Alexandrov called him "one of the brightest young mathematical talents of Europe" and credited him with mastering Noether's theories, while adding significant results of his own, and "more than anyone else, [helping] to make her ideas widely known" [Alexandroff 1935, 5]. Three years later, van der Waerden taught a highly successful course on ideal theory in Göttingen, which did much to spread awareness of Noether's work. Some of Alexandrov's most vivid memories of Emmy came from the winter semester of 1928–1929 when they were together in Moscow. He spoke of her keen interest in the Soviet experiment and firm intention to visit again, but this was not to be.

As an intimate friend with longstanding ties to the Göttingen community during the 1920s, Alexandrov left a striking portrait of Emmy Noether as he knew her. What he, of course, did not pretend to know concerned her earlier life, about which rather little, in fact, is known. Hermann Weyl fully recognized the importance of her family background, but he noted at the outset of his memorial address that he could say little about Emmy's biography. Nor had he any chance to learn more about her life now that he, like her, was separated by an ocean from their homeland. Nevertheless, he tried to paint a picture of life in Erlangen as he imagined it to have been. He offered quite vivid portraits of the two contrasting personalities who dominated mathematical life in Erlangen: Max Noether and Paul Gordan, the first Emmy's father, the second her *Doktorvater*. He even tried to convey a sense of how their generation's sense of solidity surely "prevailed in the Noether home [leading to] a particularly warm and companionable family life.". This type of atmosphere, with "its comfort and bourgeois peacefulness" was now gone forever [Weyl 1935, 429].

Weyl clearly identified the Noether family, but Emmy in particular, as "good Germans" in a generic sense. Perhaps Weyl also felt unnerved by Emmy's apparent inability to grasp evil in the world. She had lived her whole life as a fully integrated German Jew, which meant of course that anti-Semitism was no stranger to her, but when the barbarians came to power and threatened to sweep away everything she loved, she reacted not only with restraint but with an almost super-human equanimity. Those lonely months during the spring of 1933 – the time when they were last together in Göttingen – no doubt profoundly shaped Weyl's view of her, yet his opinion seems to have wavered between two extremes: Emmy was either a tower of moral strength or she was simply naive.

Although Hermann Weyl knew Wilhelmian Germany exceedingly well, he may never have met Max Noether and he probably only got to know his daughter in 1927, when he spent a semester as guest professor in Göttingen. In attempting to explore Emmy Noether's social and intellectual background in this book, we have based our interpretation primarily on contemporary documentary sources, few of which, of course, were available to Weyl in 1935. We have also tried to avoid stereotypic themes found in much of the secondary literature. Many standard studies of women in the history of mathematics have chosen to follow Weyl's lead by comparing Noether with the internationally renowned Russian mathematician Sofia Kovalevskaya, who appears in several places in this book as well. These comparisons, to be sure, rarely have anything to do with serious interest in what these women accomplished as mathematicians. Very often, they are coupled together as two trailblazers in a field then totally dominated by men, even though neither really saw herself in such a role. Talk of glass ceilings, after all, was yet to come, whereas gender roles in that era were exceedingly constrained. Earlier commentators usually could not get past the notion that a "lady mathematician" was a freak of nature, a view clearly supported by the scarce number of these creatures then walking the earth. How that has changed! Contemporary opinions of Emmy Noether – and these were quite mixed – clearly have considerable importance for understanding the context in which she lived. Even more important – especially for our undertaking – are those sources that tell us how she thought about herself and the world around her, and especially how she expressed those thoughts. Others often compared her as a mathematician with Richard Dedekind (no one writing about her mathematics would have imagined a comparison with Sofia Kovalevskaya), but she quite rightly said about herself "I always went my own way" (see the opening of Chapter 2).

We have also departed from Weyl's interpretation in another important respect. Most standard accounts of Emmy Noether's intellectual development have followed his tripartite division of her career:[8] (1) the period as a post-doc, 1907–1919, followed by (2) her work on the general theory of ideals, 1920–1926, and then (3) her contributions to non-commutative algebras, their representation theory, and applications to commutative number fields, 1927–1935 [Weyl 1935, 439]. This periodization is certainly apt and useful to a point, but it also can easily lead to quite misleading impressions. Those who have adopted it have tended to underplay the significance of the first period, while overlooking some of the threads that ran through all three phases of Noether's career.

[8] Weyl's obituary of Hilbert was somewhat similar; there he discerned that the master's work fell into five periods [Weyl 1944, 4: 135].

Emmy Noether was nearly forty years old when she began publishing the papers on modern algebra that made her famous. By the mid-1920s, she had become the leader of an international school that would soon thereafter exert a deep and lasting influence on mathematical research. All her most familiar and significant work was thus undertaken during the latter two periods, when many of her ideas and findings quickly propagated through the network of the Noether school. Little wonder, then, that this success story has completely dominated nearly all the accounts of Noether's life. Most of Noether's publications from the first period, on the other hand, received little attention during her lifetime. This applies even to her famous paper "Invariant Variational Problems" [Noether 1918b], which today is perhaps her best-known single work. As documented in [Kosmann-Schwarzbach 2006/2011], this paper was rarely ever cited until many years after Noether's death. No doubt Weyl's periodization of her research interests offers a useful schema, so long as we are not misled by it into thinking that Emmy Noether's earlier work had little to do with her publications from the 1920s. As Uta Merzbach noted, a great deal of her work had clearly identifiable classical roots:

> Her deep knowledge of the literature and her ability to recognize and bring to the fore those concepts that would prove most fruitful prepared her . . . to undertake her grand synthesis. If one examines her work after 1910, one finds continual growth, but little change in methodological pattern. [Merzbach 1983, 169][9]

This should come as no surprise if we remember that Emmy Noether had a thorough knowledge of the mathematical literature of her time; she was well-versed in major works from the latter half of the nineteenth century.

As pointed out in [Koreuber 2015, 5], Weyl's tripartite framework is highly problematic if one hopes to gain a deeper understanding of Emmy Noether's intellectual growth. While our purpose here is certainly not to probe her works in any depth, in Chapter 2 we have attempted to give a balanced picture that restores the critically important first period in her career. Those years form part of the larger context taken up in Chapter 3, which deals with her life in Erlangen, where she grew up and was long known as the daughter of the eminent mathematician Max Noether. When Emmy Noether finally left Erlangen in 1915, she did so with the hope of joining the faculty in Göttingen, a plan supported by its two senior mathematicians, Felix Klein and David Hilbert. Their efforts, however, at first failed, and as recounted in Chapter 4, it took four long years before Noether was allowed to habilitate in Göttingen. During those years, both Hilbert and Klein had become deeply immersed in problems that stemmed from Albert Einstein's novel approach to gravitation, the general theory of relativity. Noether worked first with Hilbert and then with Klein, and in [Noether 1918b] she ultimately unraveled one of the major mathematical mysteries that they and Einstein had struggled to solve,

[9] A more recent study that argues for a similar view is [McLarty 2017].

namely, the role of energy conservation in physical theories based on variational principles.

Noether's best-known works in algebra stem from the second period, when she made major contributions to ideal theory. She was almost 40 when she published "Ideal theory in ring domains" [Noether 1921b], one of her most influential algebraic publications. Here she introduced the general concept of rings satisfying the ascending chain condition, familiar today as Noetherian rings. Soon afterward, her reputation as a leading algebraist began to spread beyond Göttingen, leading to her fame as "der Noether." In describing this work from her second period in Chapter 5, we sketch the general shift from classical to modern algebra illustrated by comparing Noether's work with Richard Dedekind's earlier theory of ideals in number fields.[10]

After this, the focus in Chapters 5, 6, and 7 turns to Noether's relationships with the four other mathematicians who appear in "Diving into Math with Emmy Noether": Bartel L. van der Waerden, Pavel Alexandrov, Helmut Hasse, and Olga Taussky. While none of these four took a doctoral degree under Noether, all were closely connected with her school in one way or another. Each, in fact, represents a strand of influence that ran through the Noether school, thereby contributing to its diverse and eclectic character. Van der Waerden's principal interests, when he arrived in Göttingen from Amsterdam, were closely related to Max Noether's work in algebraic geometry. After studying under Noether's daughter and then under Emil Artin in Hamburg, he published his classical two-volume textbook *Moderne Algebra* [van der Waerden 1930/31], which for decades afterward served as the standard introduction to the subject. The Russian topologist Pavel Alexandrov was a regular visitor in Göttingen during the summer months. As one of Emmy Noether's closest friends, he spent countless hours "talking mathematics" with her, eventually joined by another topologist, Heinz Hopf. These conversations proved of vital importance for the emergence of modern topology, a field that began to take on clear form in their textbook [Alexandroff/Hopf 1935]. Both van der Waerden and Alexandrov very consciously adopted Noether's conceptual approach in writing these two seminal works, which distilled and synthesized essential knowledge in two fundamentally new disciplines: abstract algebra and algebraic topology.

During the final phase of Noether's career, Helmut Hasse was her closest collaborator. As a student of Kurt Hensel in Marburg, Hasse developed a new local-global principle based on Hensel's p-adic numbers that proved highly fertile for research in algebraic number theory. As he began to explore a new research agenda for class field theory, Emmy Noether pointed out the relevance of ongoing work on hypercomplex number systems (i.e., non-commutative algebras) for generalizing the number-theoretic investigations of Hasse and Artin. Thanks to the carefully edited publication of her letters to Hasse, published in [Lemmermeyer/Roquette 2006], one can easily recognize how Noether's ideas had a catalytic effect on Hasse's work after 1927. Her constant, unrelenting prodding,

[10] For a detailed comparison, see [Corry 2017].

mixed with praise and encouragement, played a major part in their symbiotic relationship, underscoring the importance of purely human factors in mathematical research. Noether's parallel collaboration with Richard Brauer soon led to a threesome, who together succeeded in proving the Brauer-Hasse-Noether theorem.

Noether's relationship with Olga Taussky was unlike any other, not least because she, too, was a woman with a mind of her own. Their first lengthier interactions took place during the academic year 1931/32 when Taussky came to Göttingen as a young Viennese post-doctoral student, having been hired by Richard Courant to lend help in editing Hilbert's early works on number theory. She was highly qualified to do so, having studied under Phillip Furtwängler, a leading expert on class field theory. After returning to Vienna for two years, Taussky rejoined Emmy Noether at Bryn Mawr College in 1934, a difficult time in the lives of both women, as Taussky would recall late in her life. Olga Taussky never became an enthusiast for Noether's abstract style of mathematics, and yet her encounters with Emmy, particularly during the last year of her life, proved to be of great importance for the young woman's career.

Indeed, none of these four mathematicians – van der Waerden, Alexandrov, Hasse, and Taussky – who went on to write hundreds of papers and produce dozens of Ph.D.s in the course of their careers, can really be called a disciple of Emmy Noether, even though all of them were inspired by her ideas and personality in significant ways. Just this, in fact, explains why we want to focus attention on these four individuals. By highlighting their roles within the Noether school – a community quite unlike earlier, more conventional mathematical schools – we gain a clear sense of Emmy Noether's unique leadership style, her commanding presence, as well as the open-ended nature of this highly improvisational undertaking. This said, we do not wish to overlook the quite large group of those who were Noether's students in the narrower sense, in particular her special favorite Max Deuring. He and a few others receive some attention in our book as well, even though its principal aim is to spotlight the larger dimensions of the Noether school. As Hermann Weyl emphasized, she was at the very center of Göttingen mathematics, just as the Göttingen scientific community was a manifestation of Weimar Germany's vibrant cultural life. As one of Weimar culture's leading representatives, Albert Einstein later wrote about Emmy Noether's highly significant role in this ultimately tragic story.[11]

Chapter 8 describes the traumatic events of 1933 that dramatically ended mathematical life in Göttingen as Noether had known it. She and Richard Courant, the director of the Mathematics Institute, were both forced into exile in the United States. Helmut Hasse would ultimately be appointed to Courant's chair, but while still in Marburg he initiated a campaign to maintain Noether's modest position in Göttingen. Predictably, these efforts failed, but through the intercession of friends

[11] For the interpretation of Göttingen mathematics as a phenomenon within the larger context of Weimar culture, see [Rowe 1986]. Einstein's obituary of Noether is analyzed in Section 9.1 of *Emmy Noether – Mathematician Extraordinaire* [Rowe 2020b].

in the United States, Emmy Noether gained a temporary appointment at Bryn Mawr College, a distinguished institution of higher learning for women.

In Chapter 9, we briefly recount Bryn Mawr's importance for the history of mathematics before relating various events and circumstances connected with Noether's association with the college. Emmy Noether's last two years in the United States were filled with all kinds of worries, few of which she spoke about even with her closest friends. One of these was Anna Pell Wheeler, chair of the Mathematics Department at Bryn Mawr College, who in many ways helped her to adjust to life in the United States. During her 18 months there, she also began to spread the gospel of modern algebra in weekly lectures at Princeton's Institute for Advanced Study, where her seminar attracted a number of prominent as well as up-and-coming mathematicians. Her collaborator from Germany, Richard Brauer, attended regularly, as did Nathan Jacobson. The latter filled in for Emmy Noether at Bryn Mawr the following year and would later edit her *Collected Papers* [Noether 1983]. Her sudden death on 14 April 1935, following an operation, came as a huge shock to everyone, perhaps most of all to her brother Fritz, who also had been forced to leave Germany with his wife and two sons. Emmy had tried in vain to find work for him in the United States, after which he took a position at Tomsk Polytechnic University in Western Siberia.[12]

Among earlier scholars who studied the life and work of Emmy Noether, we would like to underscore the pioneering efforts of Auguste Dick, who gathered a good deal of interesting information in the process of writing [Dick 1970/1981]. These materials can be found today in her literary estate, located in the Archive of the Austrian Academy of Sciences in Vienna, which kindly granted us permission to reproduce several photographs that appear in this volume. Here we would like to mention an exchange of letters between Dick and B.L. van der Waerden, written shortly after she interviewed him in Zurich. On 28 February 1967, he asked whether he had understood her correctly regarding one of Dick's motivations in researching Noether's life, namely to refute the then standard view that women were less capable of doing creative work in mathematics than men. Van der Waerden found this general hypothesis unconvincing, but he also thought that he could reach an objective conclusion by means of a statistical test. In her reply, written on March 4, Dick, too, thought that a statistical analysis might shed new light on this question.

By this time, van der Waerden was steeped in studies of ancient science, guided in part by a widely shared view of Greek culture. In his letter to Dick, he offered the opinion that the ancient Greeks had a special genius for philosophy, mathematics, sculpture, and the composition of dramatic works. He also believed that modern-day Jews were similarly gifted, which seemed to suggest that Emmy Noether's Jewish background accounted for her intellectual brilliance. Being well versed in statistical methods, van der Waerden set up a null hypothesis based on

[12] The subsequent fate of Fritz Noether and his family is discussed in Section 9.2 of *Emmy Noether – Mathematician Extraordinaire* [Rowe 2020b].

the assumption that during the period from 1900 to 1950 mathematical talent was equally distributed between men and women.[13] He then sent the results to Auguste Dick in a letter from March 8; these, he thought, thoroughly refuted her contention. How she responded, if at all, we do not know. In the light of what we do know today, however, van der Waerden's mathematical exercise must strike any informed observer as hopelessly naive. Given the extremely limited professional opportunities open to women who chose to study higher mathematics during the early decades of the twentieth century, it would actually seem more pertinent to wonder why a surprisingly high number of doctoral degrees were conferred on females during this period.[14] Some of these women, particularly those who had an association with Göttingen University or Bryn Mawr College, appear elsewhere in this book, though we have only occasionally touched on the larger theme of roles for women in modern mathematics.

Any contextualized account of Emmy Noether's life cannot overlook the fact that she was a woman in a world totally dominated by men. She also happened to come from a Jewish family. In her reply to van der Waerden from March 4, Auguste Dick wrote that she was interested in Emmy Noether for a number of reasons, another being her Jewish background.[15] She understood from van der Waerden's remarks that he thought this a likely reason for her extraordinary talent, and Dick tended to agree. No doubt she was much surprised to learn that this was not what he meant at all. For he replied, saying that "Emmy Noether was unique and altogether different from all other Jews that I know. You probably know that she was called 'der Noether' in Göttingen. She was motherly, without being typically feminine, just as she was not typically Jewish."[16] Van der Waerden was here invoking familiar stereotypes, perhaps even ones that Noether shared as a product of those times.

A person is considered *discriminating* if they can discern significant differences; those who cannot, on the other hand, will often *discriminate* merely on the basis of personal prejudice. Within the small mathematical elite that Emmy Noether moved in, the latter mentality was anything but rare. Although their views on social or political affairs were by no means homogeneous, Noether's peers were often no more astute when it came to such matters than were "ordinary" people. Moreover, as suggested by van der Waerden's opinions relating to gender and race, such attitudes could easily affect a person's judgment when it came to evaluating other mathematicians' creative abilities.

[13]He based this test on the assumption that roughly 20% of those who studied at European or American universities were women.

[14]In [Green/La Duke 2009] the authors found that over 14% of the doctorates awarded in the United States during the period 1900–1940 were conferred on women.

[15]Dick's interest in the Noether family led her to undertake extensive genealogical research, as can be seen from several documents in her literary estate. She also researched the careers of mathematicians in Vienna and Prague who were persecuted during the Nazi era; see [Pinl/Dick 1974].

[16]This passage was cited in the original German in [Siegmund-Schultze 2011b, 216]. The letters from van der Waerden are in Nachlass-Dick 11-35, Archive of the Austrian Academy of Sciences in Vienna.

As we have noted in various places in this book, but especially in Chapter 8, there was a strong tendency in Germany to link mathematical style with ethnicity, a viewpoint by no means restricted to anti-Semites. In essence, this stemmed from the belief that traditional Talmudic studies had sharpened the minds of *male* Jews in specific ways that promoted logical thinking and critical acumen. As a result, so many believed, they excelled especially in those mathematical disciplines in which abstract theorizing played a major role, one of the most important being abstract algebra. Since Emmy Noether was the acknowledged leader of this direction in mathematical research, her work obviously fit this stereotype (disregarding her gender). Those who felt no sympathy for abstract algebra were easily inclined to regard this new trend as somehow "Hebraic," and hence foreign to what they imagined to be good, sound "German" mathematics.

These stereotypes were particularly widespread in Germany during the postwar years of the Weimar Republic, a period during which career opportunities for Jewish mathematicians improved markedly. And although such views engendered little serious discussion before 1933, much less open debate, afterward they passed smoothly into more familiar forms of Nazi propaganda. Faced with seeing their beloved teacher banned from the German universities, Noether's faithful students countered by underscoring how her work was anchored entirely in the tradition of Richard Dedekind, one of the great German mathematicians of the nineteenth century. To follow the logic of their argument, they as good Germans wanted the government to recognize that Noether was working on behalf of "Aryan mathematics." Their efforts were in the end in vain, but the thrust of all this stemmed evidently from the conviction that "most" German Jews had a distinctly different way of thinking about mathematics, one that favored abstract theorizing while neglecting fields with close ties to the physical sciences. No one, it seems, whether then or earlier, seriously questioned that opinion by pointing out that for every Emmy Noether representing the first tendency, there was a Fritz Noether practicing the second. This larger point was brought out forcefully in the exhibition "Transcending Tradition: Jewish Mathematicians in German-Speaking Academic Culture," which presented a wide array of works by German-Jewish scholars. These completely refute the claim that there was a "typical form of 'Jewish mathematics', remote from geometrical intuition or from applications" [Bergmann/Epple/Ungar 2012, 134].[17]

A book such as this one could obviously not have been written without the efforts of many others, including those whose names appear in the works cited throughout. Rather than attempting to discharge our debt by individually listing all those who have contributed directly or indirectly to making this book possible,

[17] David Hilbert was one of the few who forthrightly claimed that "mathematics knows no races" (see [Siegmund-Schultze 2016]). His colleague, Felix Klein, thought that geometrical intuition (*Anschauung*) was deeply rooted in the Teutonic race. Yet as Klein and Max Noether well knew, after 1890 this impulse lost ground in Germany just as it was being taken up by leading Italian geometers: Corrado Segre, Guido Castelnuovo, and Federigo Enriques, all of whom were of Jewish descent.

we take this opportunity to thank everyone collectively, while singling out only a few who deserve special thanks. We must begin, first of all, by thanking the representatives of the Berlin Mathematics Research Center MATH+ for their vital support in making the interdisciplinary conference in honor of Emmy Noether possible. This was a truly unique multi-disciplinary event, one that led to many fruitful discussions and intellectual exchanges from a variety of perspectives. For those involved in its conception and realization, it proved to be a rewarding experience that also offered many new reflections with regard to Emmy Noether's past and present relevance for mathematics. Second, our thanks go to the aforementioned Sandra Schüddekopf and Anita Zieher of *portraittheater Vienna*, along with the four actors who appear in supporting roles: Alexander E. Fennon (van der Waerden), Werner Landsgesell (Alexandrov), Helmut Schuster (Hasse), and Karola Niederhuber (Taussky). Their collective efforts provided the true inspiration for this book. We are also grateful for the cooperation we received from archivists working at the Austrian Academy of Sciences, Bryn Mawr College, Caltech, Göttingen State and University Library, and Oberwolfach Research Institute for Mathematics. All provided essential help to us in procuring documents and digital images for this book. In particular, we also wish to thank Qinna Shen, Professor of German Studies at Bryn Mawr College, for her efforts in supporting this project. Thanks also to Ayse Gökmenoglu for the care she took in producing the photos included in it, and to Catriona Byrne and Rémi Lodh at Springer Nature for their helpful advice in supporting this venture.

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David E. Rowe
Mechthild Koreuber

Chapter 1

Diving into Math with Emmy Noether

Some 100 years ago a notice appeared in the journal of the German Mathematical Society that read: "Dr. Emmy Noether has habilitated as a lecturer in mathematics at Göttingen University." This quiet announcement was actually the resounding final chord in a long struggle that went on for four years and only ended on June 4, 1919, when Noether joined the Göttingen faculty. Habilitation, which long stood as a formal bulwark that prevented women from teaching at German universities, had finally fallen. For Emmy Noether, this new status represented a marvelous opportunity to pursue her mathematical interests in research and teaching without any restrictions or limitations. A distinctive approach to mathematics – one that placed abstract algebraic concepts and their mutual relationships at the center of research – would soon become Noether's credo. Her aim was to show how this approach could lead to an enrichment of mathematics, a goal she shared with the many talented young mathematicians who by the early 1920s had begun to gather around her.

Anniversaries mark an occasion to reflect on a person's life and work, whereas a theatrical production can offer the illusion of bringing someone back to life (Fig. 1.2). In our play we made use of correspondence, obituaries, and the memories of Emmy Noether's contemporaries in an attempt to approach her historical figure through documentary sources, while allowing her personality to unfold on stage. Noether's life was complicated, with many ups and downs. Despite her outstanding mathematical achievements, she experienced professional discrimination and marginalization, not to mention the racist threats that forced her to emigrate. Yet her life was also simple, because throughout it mathematics formed the center of her interests. Despite all the adversity and rejection, Emmy Noether's passion for mathematics never wavered and this guided her through an extraordinary life.

D. E. Rowe, M. Koreuber, *Proving It Her Way*, https://doi.org/10.1007/978-3-030-62811-6_1

1.1 Emmy Noether's Timeline

1882 Amalie Emmy Noether is born on March 23 as the first child of Jewish parents in Erlangen. Her father, Max Noether, is professor of mathematics at the university; her mother, Ida Noether née Kaufmann, comes from a wealthy family in Cologne. Emmy Noether will eventually have three younger brothers.

1889–1897 she attends the secondary school for girls in Erlangen.

1900 after private tutoring, she passes the Bavarian examination for French and English language teachers. Her goal, though, is not to work as a teacher, but to pursue her academic interests.

1900–1903 she attends courses in mathematics, romance languages, and history at Erlangen University, while preparing to take the qualifying exam required for entry, which she passes in 1903.

1903/04 as a guest auditor in Göttingen she attends mathematics courses taught by David Hilbert, Felix Klein, and others. By the end of the semester she becomes ill and returns home. The following year, she enters Erlangen University as a regularly enrolled student of mathematics.

1907 she graduates *summa cum laude* with a dissertation written under Paul Gordan on the invariants of a ternary biquadratic form. The work reflects her mentor's algorithmic approach to invariant theory. Later she will describe it as "calculations," "formula thickets," or even just as "crap."

1907–1915 after graduation Noether continues to work on mathematics in Erlangen. Although unpaid, she works voluntarily to support courses taught by her father and Gordan. After Gordan's retirement, she does the same under his successors, Erhard Schmidt (1910) and Ernst Fischer (from 1911). During walks and discussions with Fischer, she gains her first insights into abstract algebra.

1909 she becomes a member of the German Mathematical Society (Deutsche Mathematiker-Vereinigung) and delivers a lecture at its annual meeting in Salzburg, the first woman to do so. For years afterward she would regularly speak at these meetings, interacting with other mathematicians, and taking an active part in the discussions that followed their talks.

1913 she accompanies her father to Göttingen, where they meet with Klein to discuss plans for the obituary Max Noether will write for his colleague Paul Gordan. Klein is deeply impressed by the breadth of her mathematical knowledge.

1915 she receives an invitation from Klein and Hilbert to pursue post-doctoral studies in Göttingen. Hilbert engages her to help promote his research program in field physics based on Einstein's general theory of relativity. Noether's expertise in invariant theory makes her an ideal assistant for Hilbert's research work. With the support of the Göttingen mathematicians, she applies for certification to teach, submitting a post-doctoral (habilitation) thesis connected with Hilbert's 14th Paris problem. Her application leads to an intense controversy in the faculty, which votes to refer the matter to the Ministry of Culture. Since this case involved a fundamental question – whether a qualified woman had the right to teach – the Ministry refuses to set this precedent. Instead, a compromise is reached enabling Noether to teach courses announced under Hilbert's name.

1918 Klein submits her paper on differential invariants to the Göttingen Scientific Society. Einstein writes to Hilbert: "It impresses me that one can view these things from such a general standpoint. It wouldn't have hurt the Göttingen troops in the field if they had been sent to Frl. Noether."

1919 in the wake of the profound social and political changes in Germany after the First World War, women gain the right to teach at universities. Noether submits a new habilitation thesis, her famous paper "Invariant Variational Problems," already published in 1918. Its two main theorems show the connection between symmetries and conserved quantities in variational systems under very general conditions. One of its main applications clarifies the status of energy conservation in the general theory of relativity. Not until the 1950s, however, will theoretical physicists come to appreciate the importance of these theorems, which today bear her name.

4 June 1919 Emmy Noether presents her inaugural lecture on module theory as the final requirement before receiving the *venia legendi* and joining the Göttingen faculty. With her new status, she now begins her teaching career as a Privatdozent, a position without remuneration.

1921 she publishes one of her seminal papers, "Idealtheorie in Ringbereichen," which adopts the abstract approach for which she will become famous. Written with unusual clarity, this classical contribution to the ideal theory of "Noetherian rings" can still be read today by students wishing to learn key concepts in abstract algebra. Over the next several years, ideal theory would remain her principal field of research.

1922 Noether receives the honorary title of associate professor, a distinction signifying recognition, but without any financial benefits.

1923 she gains for the first time a teaching appointment that provides a small remuneration to support her scientific work. Although living very modestly after her father's death in 1921, she finds herself in dire economic straits due to the rampant hyperinflation.

1926 Grete Hermann takes her doctoral degree as the first official graduate of the "Noether school." In addition to Hermann, Noether's pupils include Max Deuring, Heinrich Grell, Jakob Levitzki, and Kenjiro Shoda. A number of prominent foreign mathematicians, in particular Bartel L. van der Waerden from Amsterdam and Pavel Alexandrov from Moscow, come to Göttingen around this time. A close, family-like atmosphere pervades the fledgling "Noether school," composed of young mathematicians eager to learn about abstract algebra and its applications to other mathematical disciplines.

1928/29 Noether accepts an invitation from Alexandrov to spend a semester in Moscow as a visiting professor. She offers a lecture course on modern algebra attended by 20-year-old Lev Pontryagin, who became one of the century's leading topologists. On returning to Göttingen, she speaks positively about social developments in the Soviet Union. Her sympathy for the Bolshevik experiment soon earns her the reputation of being a Marxist.

1929 although initially viewed with skepticism by some of her Göttingen colleagues, Noether's research and that of her students reveals the fruitfulness of her methodological approach. A highly communicative oral style becomes her trademark and her lecture courses take on programmatic significance. Other young mathematicians – including Helmut Hasse, Gottfried Köthe, and Chiungtze Tsen – take up her ideas and develop them further.

1930 she teaches for one semester in Frankfurt, filling in for Carl Ludwig Siegel during his leave of absence. One of her students there is Paul Dubreil, later a member of the Bourbaki Circle. He and his wife, Marie-Louise Dubreil-Jacotin, attend Noether's lectures in Göttingen. They both go on to successful itinerant careers in France, where Dubreil-Jacotin becomes the first female professor of mathematics.

1932 Noether receives the Ackermann–Teubner Memorial Prize together with Emil Artin. At the International Congress of Mathematicians in Zurich she gives a keynote lecture on hypercomplex systems in relation to commutative algebra and number theory. Despite such distinctions and international recognition, she is never seriously considered for a professorship.

25 April 1933 she and several of her colleagues in mathematics are persecuted by the Nazi regime, which enacts a Law for Restoration of the Civil Service. Although she was never a civil servant, she is placed on temporary leave while her case is under review. Colleagues and students try to support her with testimonials and petitions, but on September 2, 1933 her teaching license is revoked on the grounds of her Jewish ancestry.

1933 she receives an invitation from Somerville College in Oxford, but also an offer for a visiting professorship at Bryn Mawr College in Pennsylvania. When the financial arrangements in Oxford remain cloudy, she accepts the offer from Bryn Mawr.

1934 in addition to her courses at Bryn Mawr, Noether gives weekly lectures on topics in algebra at the nearby Institute for Advanced Study in Princeton. There she encounters several familiar figures, fellow émigrés like Hermann Weyl and his assistant, Richard Brauer. In the summer Noether travels to Germany and visits her brother Fritz, who will soon emigrate to the Soviet Union. She lectures in Hamburg and disposes of her household belongings in Göttingen before returning to Bryn Mawr. There she is joined by another émigré, Olga Taussky, who becomes part of a new circle of "Noether girls."

14 April 1935 Emmy Noether dies from complications following an operation for the removal of a fibroid. Her ashes are placed in an urn and buried under the cloisters of the M. Carey Thomas Library on the campus of Bryn Mawr College.

Figure 1.1: Anita Zieher as Emmy Noether, Courtesy of Portraittheater Vienna

(a) Emmy Noether, ca. 2020

(b) Emmy Noether, ca. 1930

Figure 1.2: Emmy Noether and her Doppelgängerin, Portraittheater Vienna and Emmy Noether Papers, Bryn Mawr College Special Collections

1.2 Four Mathematicians from Noether's Circle

Bartel L. van der Waerden (1903–1996) was a Dutch mathematician whose research interests ranged from abstract algebra and algebraic geometry to quantum mechanics. In 1919 he began studying mathematics in Amsterdam; one of his teachers, L.E.J. Brouwer, recommended him to Emmy Noether. During his first stay in Göttingen in 1924/25, he learned modern algebra from her. Inspired by these studies, he applied algebraic methods in his doctoral dissertation on the foundations of enumerative geometry. He then returned to Göttingen on a Rockefeller fellowship, which also took him to Hamburg. There he and Emil Artin considered writing a textbook on modern algebra, which van der Waerden later published on his own. After spending four years in Groningen, van der Waerden was appointed professor of mathematics in Leipzig in 1931. After the war, he held a professorship in Amsterdam and then at the University of Zurich, where he taught from 1951 until his retirement in 1973.

With the publication of his two-volume textbook *Moderne Algebra* in 1930–31, van der Waerden made a decisive contribution to the canonization of abstract algebra. Based in part on lectures by Noether and Artin, these volumes played a major role in promoting a deeper understanding of and appreciation for the new structural approach to mathematics.

Pavel Alexandrov (1896–1982) was a Soviet mathematician who did pioneering work in topology. During his studies at Lomonosov University in Moscow he met Pavel Urysohn, and they soon became intimate friends. In 1923 they traveled to Göttingen, where they interacted with several mathematicians, in particular

Emmy Noether. Their passionate enthusiasm for topology matched well with her equally strong interest in new abstract directions in mathematics.

Following Urysohn's tragic death in 1924, Alexandrov visited Göttingen often to discuss mathematics with Noether. Her ideas helped stimulate the development of algebraic topology, as reflected in *Topologie* (1935), the classic text by Alexandrov and Heinz Hopf. Alexandrov spent the academic year 1927/28 in Princeton, and in 1928 he was named corresponding member of the Göttingen Academy of Sciences. At his invitation, Noether spent the winter semester of 1928/29 in Moscow. Her inspiring lectures on abstract algebra were followed there with great interest. Soon after she returned to Göttingen, Alexandrov was appointed to a professorship at Lomonosov University, but once the Nazi government came to power he no longer traveled to Germany.

Helmut Hasse (1898–1979) was a German mathematician who specialized in algebraic number theory and abstract algebra. After WWI he began his studies in Göttingen, but left in 1920 to study under Kurt Hensel in Marburg. He taught in Kiel before his appointment to a professorship in Halle in 1925. In 1930, Hasse succeeded Hensel in Marburg and four years later he was appointed to the professorship in Göttingen formerly held by Hermann Weyl before he resigned. Despite the fact that she was living in exile, Emmy Noether congratulated him on this appointment and expressed her hopes that Göttingen would remain an important mathematical center. After the war, the British authorities in Göttingen decided that Hasse could not be reappointed in view of his past support of the Nazi regime. He thereafter accepted a research position at the Academy of Sciences in East Berlin, and in 1949 took on a professorship at the Humboldt University. One year later, he left this position to accept a chair at the University of Hamburg, where he remained until his retirement in 1966.

Hasse made fundamental contributions to algebraic number theory, particularly class field theory and the theory of algebras. He and Noether engaged in an intensive correspondence that clearly conveys Noether's ability to inspire creative mathematical exchanges. In 1932, Hasse, Noether, and Richard Brauer published one of the central theorems in the theory of algebras.

Olga Taussky (1906–1995) was an Austrian-American mathematician whose early work centered on algebraic number theory. After taking her doctorate in Vienna under Philipp Furtwängler, Richard Courant invited her to Göttingen in 1931 so that she could assist with the editing of the first volume of David Hilbert's Collected Works. During this stay she attended Emmy Noether's lectures and struck up a friendship with her. In 1934 Taussky was awarded a scholarship from Bryn Mawr College, which gave her the chance to work with Noether again.

After Emmy's Noether's death in 1935, Taussky became a Fellow at Girton College in Cambridge University. She married the Irish mathematician, Jack Todd, and both undertook war-related research before coming to the United States in