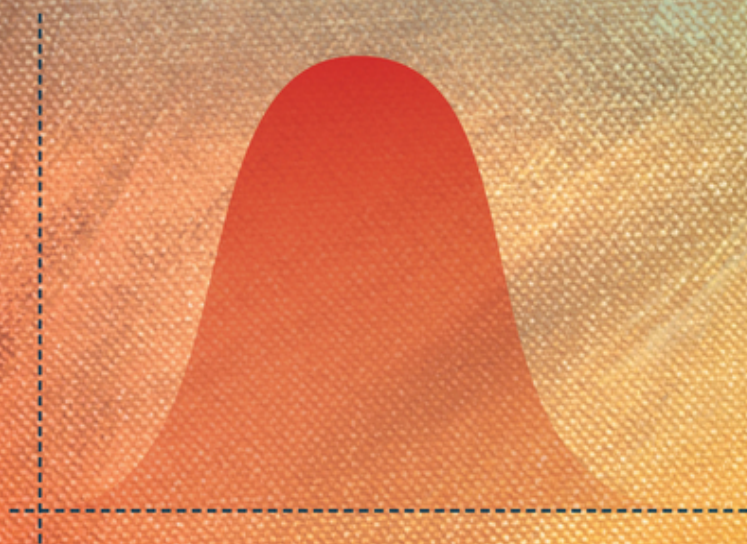


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# **VOLATILITY**

**PRACTICAL OPTIONS THEORY**



**ADAM S. IQBAL**

**WILEY**



# Volatility

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# Volatility

*Practical Options Theory*

ADAM S. IQBAL

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*To Mum, Dad, my wife, Gosia, and my beautiful  
one-year-old daughter, Maria, without whom this  
book would have been finished up to one year earlier.*





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# Preface

**T**his book studies options, the financial contracts that provide exposure to volatility. It has one main objective.

While there already exist several excellent references on the mathematical theory underlying options (Shreve, 2000, Duffie, 2001, and Bjork, 2009) are examples among a list too long to complete here), there is a relative absence of texts that attempt to explain how to bring the many theoretical ideas into practice. The main objective in this book is to provide an intuitive, as well as technical, understanding of both the basic and advanced ideas in options theory, with the aim of encouraging translational work from theory into practical application by market makers, portfolio managers, investment managers, risk managers, traders, and other market practitioners.

I show the reader that several of the most important concepts in options theory such as implied volatility, delta hedging, time value, and many of the so-called *option Greeks* can be understood by appealing to intuitive economic arguments alone, without the need to build a formal mathematical model. Once this knowledge is in place, I go on to explain the seminal Black-Scholes-Merton mathematical model. The reader will understand how the model-free approach and mathematical models are related to each other, their underlying theoretical assumptions, and their implications to a level that facilitates their practical implementation.

The approach taken in this book may prove valuable to options traders and other practitioners tasked with making pricing or risk management decisions in an environment where time constraints mean that simplicity and intuition are of greater value than mathematical formalism. This approach may also prove useful to academics interested in the translational process of theoretical options pricing into practical application, and in the feedback loop between academia and practice.

The majority of the concepts in this text are applicable to options on equities, bonds, and commodities. However, this book provides in-depth insight into the theoretical and practical function of the FX options over-the-counter (OTC) market. Given the liquidity in FX spot (and

forwards) markets and the lower number of trading constraints (such as short selling; selling EUR-USD is the same as buying USD-EUR), FX markets arguably provide one of the better opportunities to implement the theoretical ideas presented here.

The target audience for this book includes, but is not limited to, industry practitioners, finance and economics advanced undergraduate and graduate students, MBA students, and academics interested in translational finance. I presume some exposure to undergraduate level probability, statistics, and calculus. However, this should not deter readers with less exposure to these topics for at least two reasons. First, the presentation is consciously informal with the aim of exposing ideas in their simplest form before going back and understanding their foundations. Second, I provide several mathematical appendices to assist such readers and to keep this book self-contained with respect to the most important concepts. Although at times it may not seem to be the case, the challenges in successful options trading are conceptual rather than mathematical.



# Acknowledgments

**T**his book owes a debt to the many colleagues in FX options I have worked with over my years at Goldman Sachs and Barclays Investment Bank for their engagement in our discussions about options theory and for all that they have taught me.

I also thank the people who have taken time to provide comments, proofreading, and feedback. In particular, I thank Kimiya Minoukadeh, Mobeen Iqbal, and Christopher Cullen for their efforts. All remaining errors are my own.



## About the Author

**Adam S. Iqbal** is a Managing Director and Global Head of FX Exotics and Correlation at Goldman Sachs, where he has also served as EMEA Head of G10 FX Options Trading, running several FX vanilla and exotic long-, and short-dated options portfolios. Previously, he was an FX Volatility Portfolio Manager at Pimco. He has also worked as a vanilla and exotic options trader at Barclays Investment Bank in London. He holds a PhD in financial mathematics and financial economics from Imperial College London, an MSc in applied mathematics from Oxford University, and an MSci, MA, and BA in theoretical physics from Cambridge University. Dr. Iqbal has held a visiting academic position at Imperial College London and has guest lectured at the London School of Economics. He lives in London with his wife, Malgorzata, and their one-year-old daughter, Maria.



# Volatility