

Second edition

 **Minitab® 16**  
Statistical Software



# Six Sigma

Quality Improvement  
with Minitab

G. Robin Henderson

 **WILEY**



# Six Sigma Quality Improvement with Minitab



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Second Edition

**G. Robin Henderson**

*Halcro Consultancy, Midlothian, UK*



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*To Fiona and Iain*





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# Foreword

If it were possible to add up, on a global basis, all of the benefits that organisations have experienced as a result of deploying Six Sigma techniques, the result would be truly staggering. The place of Six Sigma as an effective methodology for improving quality and performance is very well established.

Six Sigma is often introduced to organisations through training. Six Sigma practitioners gain personal development by attending a spectrum of courses from introductory sessions to expert level that is normally referred to as Six Sigma Black Belt. I personally have led many hundreds of practitioners through this process, and whilst I hope my interventions have been successful, I am only too aware that there is a limit to what participants can be expected to absorb in a classroom.

This is where Robin Henderson's book becomes truly invaluable. As well as providing students new to Six Sigma with a very readable and easy to understand introduction, this publication serves as comprehensive consolidation for those already trained. Furthermore this book extends the knowledge gained by recognised experienced practitioners.

Those looking for relevant and modern case studies from both service and manufacturing environments will be most satisfied to find them in abundance throughout the following pages. Robin Henderson demonstrates the wide applicability and power of these methods with an impressive collection of analyses and improvements drawn from his broad experience of working as a consultant as well as an academic. The combination of many chapter-end exercises, follow up activities, and the accompanying web site, form a wealth of extremely useful resources.

The success of Six Sigma would not have been realised had it not been for the development of statistical software such as Minitab. Minitab brings techniques to all of us that previously were only the domain of statisticians. Robin Henderson's book complements other well-known texts by taking the theory and explaining how to implement these methods in real situations through the use of Minitab software. Starting from a gentle introduction to Minitab, Robin builds our knowledge through detailed yet friendly explanations, and as we practice, gradually leads us on to tackle more sophisticated techniques with confidence.

I have spent much of my career working in the field of quality and performance improvement, utilising these tools and techniques, and teaching the subject to others. Since its publication, I have regularly turned to Robin Henderson's first edition of Six Sigma Quality Improvement with Minitab to check my understanding, to learn a bit more, and to find a way of explaining to my teams, a challenging concept in a straightforward way.

Now we have the benefit of this second edition that keeps us up to date on the latest developments within the Minitab tool and its brand new features. For example, Robin Henderson introduces the new Minitab Assistant that guides users through an analysis process.

There is no doubt that Robin Henderson has helped all of us in our endeavours to improve quality as author of various papers on the subject and through his involvement with the Six Sigma Study Group and the Quality Improvement Section of the Royal Statistical Society.

I warmly welcome the 2nd edition of Six Sigma Quality Improvement with Minitab and thoroughly recommend it to both new students and experienced practitioners of Six Sigma methodologies.

**Colin Barr BSc (Hons), Six Sigma Black Belt**

*Colin Barr is the founder of Colin Barr Associates, providers of training and consultancy in Business Improvement using techniques such as Lean and Six Sigma. He is also founder of Stratile, where he developed FocalPoint, a web based strategy and performance management system. Colin Barr gained a BSc (Hons) Physics from Strathclyde University, and has held director level positions at DEC and Motorola where he was trained as a Six Sigma Black Belt.*

# Preface

## Rationale

The Statistics Division of the American Society for Quality defines *statistical thinking* (<http://www.asqstatdiv.org/stats-everywhere.htm>, accessed 29 January 2011) as a philosophy of learning and action based on three principles:

- All work occurs in a system of interconnected processes.
- Variation exists in all processes.
- Understanding and reducing variation are key to success.

In a paper entitled ‘Six-Sigma: the evolution of 100 years of business improvement methodology’, Snee (2004) states that ‘The three key elements to statistical thinking are process, variation and data’ and that ‘Statistical thinking enhances the effectiveness of the statistical methods and tools’. He describes Six Sigma\* as a strategy and methodology for the deployment of statistical thinking and methods within an organization. This book aims to explain some of the most important statistical methods and demonstrate their implementation via the statistical software package Minitab® (Release 16). Minitab® and the Minitab logo are registered trademarks of Minitab, Inc. There are many excellent texts available on statistical methods for the monitoring and improvement of quality. In writing this book the author set out to complement such texts by providing careful explanation of important statistical tools coupled with detailed description of the use of Minitab, either to implement the statistical tools or as an aid to understanding them.

In *Six Sigma Beyond the Factory Floor*, Hoerl and Snee (2005, p. 23) wrote:

Another reason Six Sigma has been effective is the general availability of user-friendly statistical software that enables effective and broad utilization of the statistical tools. The statistical software package most widely used in Six Sigma is Minitab. . . . Prior to the availability of such user-friendly software, statistical methods were often the domain of professional statisticians, who had access to, and specialized training in, proprietary statistical software. Specialists in statistical methods have an important role to play in Six Sigma, but practitioners who are not professional statisticians do the vast majority of statistical applications.

The author believes that his book will be of value to such practitioners and to people involved in quality improvement strategies other than Six Sigma, to students of quality improvement and indeed to anyone with an interest in statistical methods and their implementation via software.

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\* Six Sigma is a registered trademark and service mark of Motorola Inc.

## Content

Among the features of the book are the following:

- Exposition of key statistical methods for quality improvement – data display, statistical models, control charts, process capability, process experimentation, model building and the evaluation of measurement processes.
- Detailed information on the implementation of the methods using Minitab with extensive use of screen captures.
- Demonstration of facilities provided by Minitab for learning about the methods and the software, including the new Assistant.
- Use of random data generation in Minitab to aid understanding of important statistical concepts.
- Provision of informative follow-up exercises and activities on each topic.
- No prior knowledge of statistical methods assumed.
- No prior knowledge of Minitab assumed.
- Access to Release 16 of the Minitab software is essential.
- An associated website providing data sets for download and answers and notes for the follow-up exercises.

There are eleven chapters and four appendices. In addition to the topics covered in the first edition, this edition includes new material on Pareto charts, cause-and-effect diagrams, the multivariate normal distribution, acceptance sampling, time-weighted and multivariate control charts, tolerance intervals, Taguchi experimental designs, comparison of measurement systems, analysis of categorical data and logistic regression. It also includes material on new features provided in Release 16 of Minitab such as the Assistant. A brief summary of the content of each chapter is as follows:

- Chapter 1 introduces the structured approach to quality improvement provided by Six Sigma via DMAIC – define, measure, analyse, improve and control. It outlines the role of statistical methods in Six Sigma and the capabilities of Minitab for their implementation.
- Chapter 2 provides an introduction to data display, and to Minitab and its features. It also addresses data input, output, storage and manipulation.
- Chapter 3 contains further material on the display and summary of data – exploratory data analysis techniques and techniques for use with multivariate data are introduced. Pareto charts and cause-and-effect diagrams are explained.
- Chapter 4 is devoted to fundamentals of probability and to univariate statistical models for measurements and counts. A brief introduction to the multivariate normal distribution is given and key results concerning means and proportions are presented. An



introduction to the application of discrete probability distributions in acceptance sampling is provided.

- Chapter 5 gives a comprehensive treatment of control charts and their application. Shewhart, exponentially weighted moving average (EWMA), cumulative sum (CUSUM) and multivariate control charts are covered. Reference is made to the dangers of tampering with processes and to feedback adjustment.
- Chapter 6 addresses the assessment of process capability via capability indices and sigma quality levels. Tolerance intervals are introduced.
- Chapter 7 deals with process experimentation involving a single factor and essentially addresses the question of whether or not process changes have led to improvement. The question is addressed via statistical inference and estimation.
- Chapter 8 extends the ideas introduced in the previous chapter to process experimentation involving two or more factors. Fundamental aspects of design of experiments are introduced together with the powerful features provided in Minitab for experimental design and the display and analysis of the resulting data. Taguchi experimental designs are introduced.
- Chapter 9 utilizes concepts from previous chapters in order to evaluate the performance of measurement processes for both continuous measurement and attribute measurement scenarios. Reference is made to the comparison of measurement systems.
- Chapter 10 is concerned with model building using simple and multiple regression. Response surface methodology and regression modelling with categorical response variables are introduced.
- Chapter 11 concludes the book by looking at ways in which Minitab can assist the user to learn more about the software and the statistical tools that it implements. An introduction to Minitab macros is provided.

## Using the book

This is not a book to be read in an armchair! The author would encourage users to follow the Minitab implementation of displays and analyses as he/she reads about them and to work through the supplementary exercises and activities at the end of each chapter. All but the very smallest data sets referred to in the text will be available on the website [http://www.wiley.com/go/six\\_sigma](http://www.wiley.com/go/six_sigma) in the form of Minitab worksheets or Microsoft Excel™ workbooks. It is recommended that you download the files and store them in a directory on your computer. Some of the data sets are real, others have been simulated (using Minitab!) to provide appropriate illustrations. Many of the simulated data sets are set in the context of quality improvement situations that the author has encountered. The website will also provide specimen solutions to, and comments on, the supplementary exercises.

The needs of readers will differ widely. It is envisaged that many will find the first four or five chapters sufficient for their needs. It is important to note that although a brief introduction to the Help facilities will be given in Chapter 2, many readers might find it helpful to read the

first section of Chapter 11 immediately. The reference to Help has been encountered in Chapter 2 in order to obtain more comprehensive information facilities that are available.

The reader might wonder why the chapter on control charts is before the one on measurement process evaluation, whilst in DMAIC the order appears to be reversed. The author has endeavoured to order the chapter topics in a sequence that is logical from the point of view of the development of understanding of the applied statistics. For example, one cannot fully understand a gauge R&R measurement process evaluation without knowledge of analysis of variance for data from a designed experiment. Designed experiments are usually associated with the improve phase. Indeed, control charts may be of value during all four of the measure, analyse, improve and control phases of a Six Sigma project. Each chapter will give an indication of the relevance of its content to the DMAIC sequence that lies at the heart of Six Sigma.

There is always a danger that statistical software will be used in black box fashion with unfortunate consequences. Thus the reader is exhorted to learn as much as he/she possibly can about the methods and to take every opportunity to learn from successful, sound applications by others of statistical methods in quality improvement, whether on Six Sigma projects or as part of other strategies.

It is the author's earnest hope that, through using this book, you the reader will acquire understanding of statistical methods for quality improvement and Six Sigma, skill in the application of the Minitab software, and appreciation of just how easy it is to use and of all that it has to offer.

# Acknowledgements

Grateful appreciation of help and encouragement is due to the following people: Sandra Bonellie, Colin Barr, Gary Beazant, Isobel Black, Roland Caulcutt, Shirley Coleman, Lorraine Daniels, Ross Davies, Martin Dennis, Jeff Dodgson, Geoff Fielding, Alan Fisher, Wendy Ford, Martin Gibson, Mary Hickey, Iain Henderson, Kevin Hetzler, Tom Johnstone, Graham Leigh, Ron Masson, Bill Matheson, Deborah Macdonald, Mark McGinnis, Gillian Mead, Charles Moncur, Douglas Montgomery, Bill Munro, David Panter, Gillian Raab, David Roberts, David Reed, Fiona Reed, Anne Shade, John Shrouder, William Woodall and Andrew Vickers.

Robert Raeside deserves a major thank-you for all that I learnt from him during shared involvement in the development and delivery of many training courses on quality improvement when we were both members of the Applied Statistics Group at Edinburgh Napier University. Another major influence was John Shade, a statistician with vast experience of quality improvement, with whom it was a pleasure to work, in a highly stimulating environment at Good Decision Ltd.

At John Wiley & Sons, Inc., Richard Davies, Heather Kay, Ilaria Meliconi and Prachi Sinha-Sahay have been a pleasure to work with. The author is most grateful to the copy editor Richard Leigh for his highly professional contribution to the project. Abhishan Sharma at Thomson Digital was most helpful during the typesetting phase of the project. Support from Minitab Inc. has been excellent. In particular, the author wishes to acknowledge the help of Austin Davey in the UK and of Eugenie Chung and Linda Holderman in the USA. Portions of the input and the output contained in this book are printed with permission of Minitab, Inc. Use of data sets included with the Minitab software is gratefully acknowledged. Grateful thanks are due to authors and publishers who have given permission for use of data and material. Specific acknowledgements are made in the text.

Finally, in preparing this second edition my wife Anne has been ignored once again for many, many hours – yet her support, as ever, has been immense.



# About the Author

Having studied mathematics and physics at the University of Edinburgh, Robin Henderson embarked on a 35-year career in education. For much of that time he was employed in what is now Edinburgh Napier University, teaching mathematics and statistics, at all levels.

His interest in statistics for quality improvement grew during the 1980s, largely due to involvement with colleagues Professor Robert Raeside and Professor Ron Masson in providing training courses, including courses to prepare engineers to sit the Certified Quality Engineer examinations of the American Society for Quality, and consultancy for local organizations, particularly microelectronics companies. This interest led him to leave the University in 1998 in order to work as a statistical consultant for Good Decision Ltd, Dunfermline, where he was heavily involved in the development and delivery of training courses for industry in process monitoring and adjustment, measurement process evaluation and design and analysis of multifactor experiments.

His interest in and enthusiasm for Minitab also developed during the 1980s when it began to be used at Edinburgh Napier University in the teaching of students on a wide variety of courses. Release 7 of the software, with line-printer graphics, was a far cry from the sophistication of the current version!

Since 2001, Robin has been operating as a sole consultant, trading as Halcro Consultancy, Loanhead, providing training and consultancy in statistics for quality improvement and Six Sigma. He has assisted Colin Barr Associates with the training of Six Sigma Black Belts and with statistical consultancy projects. In 2009 Edinburgh Napier University received the Queen's Anniversary Prize, the highest accolade that can be conferred on a higher or further education institution in the UK, for its pioneering research in innovative construction techniques to improve insulation in new build homes. The author is very pleased to have provided the spin-out company, Robust Details Ltd., that was created to oversee uptake of the construction solutions stemming from the research, with training in both statistical methods and Minitab.

He is also currently employed as coordinator at the Royal Infirmary of Edinburgh for the Scottish National Stroke Audit. On this project he has introduced the use of Shewhart control charts for monitoring aspects of the processes involved in the care of stroke patients. Since the first edition was published he has been principal author of two papers on healthcare applications of control charts and co-author of a paper on the technical details of estimation of process variability. He has acted as secretary to both the Committee of the Quality Improvement Section and the Six Sigma Study Group of the Royal Statistical Society, of which he is a Fellow. In these roles he was responsible for collating the views of colleagues on the draft international standards BS ISO 13053-1/2 *Quantitative methods in process improvement – Six Sigma – Part 1: DMAIC methodology, Part 2: Tools and techniques* and for subsequently preparing the comments submitted by the Society on the drafts. He is a member of ENBIS, the European Network for Business and Industrial Statistics.



# 1

## Introduction

Six Sigma is a strategic approach that works across all processes, products and industries. (Snee, 2004, p. 8)

### Overview

In an overview of Six Sigma, Montgomery and Woodall (2008) state:

Six Sigma is a disciplined, project-oriented, statistically based approach for reducing variability, removing defects, and eliminating waste from products, processes, and transactions. The Six Sigma initiative is a major force in today's business world for quality and business improvement. Statistical methods and statisticians have a fundamental role to play in this process.

This book is about the understanding and implementation of statistical methods fundamental to the Six Sigma and other approaches to the continuous improvement of products, processes and services. Release 16 of the widely used statistics package Minitab is used throughout. Mindful that in the vast majority of situations those applying statistical methods in quality improvement and Six Sigma are not statisticians, information on the implementation of each method covered will be preceded by some background explanation, typically employing small data sets. The role of each method within the define–measure–analyse–improve–control (DMAIC) framework of Six Sigma will be highlighted.

This chapter deals with quality and quality improvement and in particular with the highly successful Six Sigma approach to quality improvement. It describes the role of statistical methods in quality improvement and Six Sigma and outlines how Minitab can be used to implement these methods.

## 1.1 Quality and quality improvement

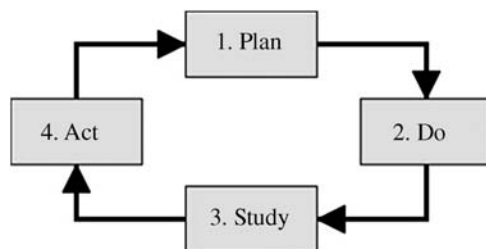
Definitions of quality abound. Wheeler and Chambers (1992, p. xix) define quality as being ‘on-target with minimum variance’. They state that operating ‘on-target’ requires a different way of thinking about processes and that operating with ‘minimum variance’ can only be achieved when a process is behaving in a reasonably stable and predictable way. Wheeler and Poling (1998, p. 3) state that continual improvement requires a ‘methodology for studying processes and systems, and a way of differentiating between the different types of variation present in processes and systems’. They refer to the cycle of activities involved in continual improvement as the plan–do–study–act (PDSA) cycle – see Figure 1.1 (Wheeler and Poling, 1998, p. 5), reproduced by permission of SPC Press, Inc. This cycle of activities is often referred to as the Shewhart–Deming cycle in honour of two key figures in the development of quality improvement methodology, Dr Walter A. Shewhart and Dr W. Edwards Deming.

In 1988 the author had the pleasure, along with some 400 others, of attending a seminar for statisticians by the late Dr Deming at the University of Nottingham. In his presentation handout he described the cycle as follows (Deming, 1988, p. 33):

- *Plan* – Plan a change or test, aimed at improvement.
- *Do* – Carry it out, preferably on a small scale.
- *Study* – Study the results. What did we learn?
- *Act* – Adopt the change or abandon it or run through the cycle again, possibly under different environmental conditions.

In his book *Out of the Crisis*, Deming (1986, p. 23) listed *14 Points for Management*, including the following:

- *Constancy of purpose* – create constancy of purpose for continual improvement of products and service.
- *Improve every process* – improve constantly and forever every process for planning, production and service.
- *Eliminate targets* – substitute aids and helpful supervision; use statistical methods for continual improvement.



**Figure 1.1** The Shewhart–Deming PDSA cycle.



Thus the Deming philosophy of quality improvement advocates never-ending continual improvement of all processes within an organization. The improvement process itself should be structured and employ statistical methods. 'Improvement nearly always means reduction of variation' (Deming, 1988).

## 1.2 Six Sigma quality improvement

Six Sigma is one of a number of quality improvement strategies based on the Shewhart–Deming PDSA cycle. Truscott devotes a chapter to comparison of Six Sigma with other quality improvement initiatives. He puts Six Sigma in perspective earlier in his book as follows (Truscott, 2003, p. 1):

Six Sigma focuses on establishing world-class business-performance benchmarks and on providing an organizational structure and road-map by which these can be realized. This is achieved mainly on a project-by-project team basis, using a workforce trained in performance-enhancement methodology, within a receptive company culture and perpetuating infrastructure. Although particularly relevant to the enhancing of value of products and services from a customer perspective, Six Sigma is also directly applicable in improving the efficiency and effectiveness of all processes, tasks and transactions within any organization. Projects are thus chosen and driven on the basis of their relevance to increased customer satisfaction and their effect on business-performance enhancement through gap analysis, namely, prior quantitative measurement of existing performance and comparison with that desired.

Six Sigma originated at Motorola Inc. as a long-term quality improvement initiative entitled 'The Six Sigma Quality Program'. (Six Sigma® is a registered trademark and service mark of Motorola Inc.) It was launched by the company's chief executive officer, Bob Galvin, in January 1987 with a speech that was distributed to everyone in the organization. In the speech Galvin reported on many visits to customers in the previous six months during which desires were expressed for better service from Motorola in terms of delivery, order completeness, accurate transactional records etc. The customers had also indicated that, with better service and an emphasis on total quality, Motorola could expect an increase of between 5% and 20% in future business from them. He therefore challenged employees to respond urgently to make the necessary improvements, emphasized the leadership role of management in the implementation of the programme and announced that Motorola's corporate quality goal had been updated accordingly. The goal included the objective 'Achieve Six Sigma capability by 1992' (Perez-Wilson, 1999, p. 131).

In addition to being a strategy for the continual improvement of quality within organizations, Six Sigma indicates a level of performance equating to 3.4 nonconformities per million opportunities, a level which some regard as being 'world-class performance'. This often leads to confusion. In this book the phrase 'sigma quality level' will be used for this indicator of process performance, as advocated by Breyfogle (2003, p. 3). Thus a sigma quality level of 6 equates to 3.4 nonconformities per million opportunities. The link between sigma quality level and number of nonconformities per million opportunities will be explained in detail in

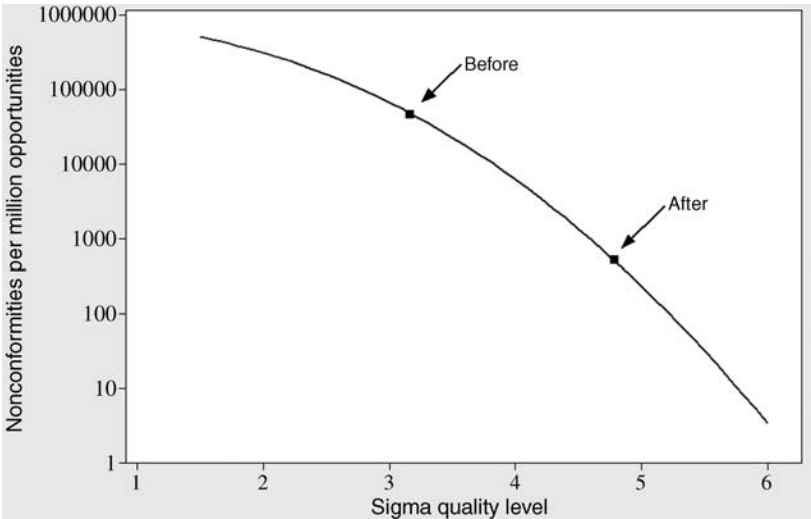
**Table 1.1** Sigma quality levels and nonconformities per million opportunities.

Sigma quality level	Nonconformities per million opportunities
1.5	501 350
2.0	308 770
2.5	158 687
3.0	66 811
3.5	22 750
4.0	6 210
4.5	1 350
5.0	233
5.5	32
6.0	3.4

Chapter 4. Table 1.1 gives some sigma quality levels and corresponding numbers of nonconformities per million opportunities.

The plot in Figure 1.2 (created using Minitab) shows nonconformities per million opportunities plotted against sigma quality level, the vertical scale being logarithmic. Many authors refer to defects rather than nonconformities and state that a sigma quality level of 6 equates to 3.4 defects per million opportunities or 3.4 DPMO.

Imagine a bottling plant where there was concern over the number of bottles of whisky containing contaminant particles. A bottling run of 14 856 bottles had yielded 701 nonconforming bottles in terms of contamination. This corresponds to  $(701/14\,856) \times 10^6 = 47\,186$  nonconforming bottles per million. Since 47 254 lies between 22 750 and 66 811, Table 1.1 indicates a sigma quality level between 3.0 and 3.5. The more comprehensive table in



**Figure 1.2** Sigma quality levels and nonconformities per million opportunities.

Appendix 1 gives the sigma quality level as 3.17 (47 461 being the entry in the table closest to 47 186).

The main source of contamination was found to be the wax-coated corks used to seal the bottles. Dialogue with the supplier of the corks led to a trial with corks produced using a new method for application of the wax. For the trial there were 8 nonconforming bottles out of 15 841. The reader is invited to verify that this corresponds to 505 nonconforming bottles per million and a sigma quality level of 4.79. Thus the higher the sigma quality level for a process the better is its performance. The points corresponding to the situations before and after the process change are indicated on the curve in Figure 1.2. Montgomery and Woodall (2008) state that ‘the 3.4-ppm metric, however, is increasingly recognized as primarily a distraction; it is the focus on reduction of variability about the target and the elimination of waste and defects that is the important feature of Six Sigma’.

Another source of confusion arises from the use of  $\sigma$ , the lower-case Greek letter sigma, as a statistical measure of variation called standard deviation. It will be explained in detail in Chapter 2. Bearing in mind Deming’s comment that ‘Improvement nearly always means reduction of variation’, the consequence is that improvement often implies a reduction in  $\sigma$ . Thus it is frequently the case that improvement corresponds to an increase in sigma quality level on the one hand, and to a decrease in sigma on the other. Therefore it is essential to make a clear distinction between both uses of sigma.

Larry Bossidy, CEO of Allied Signal led the implementation of Six Sigma with his organization (Perez-Wilson, 1999, p. 270). In June 1995 Bossidy addressed the Corporate Executive Council of the General Electric Corporation (GE) on Six Sigma quality at ‘one of the most important meetings we ever had’ (Welch, 2001). The Council was impressed by the cost saving achieved at Allied Signal through Six Sigma and, as an employee survey at GE had indicated that quality was a concern for many GE employees, Welch ‘went nuts about Six Sigma and launched it’ (Welch, 2001). One of the first steps GE took in working towards implementation of Six Sigma was to invite Mikel Harry, formerly a manager with Motorola and founder of the Six Sigma Academy, to talk to a group of senior employees. In a four-hour presentation ‘he jumped excitedly from one easel to another, writing down all kinds of statistical formulas’. The presentation captured the imagination of Welch and his colleagues, the discipline of the approach being of particular appeal to engineers. They concluded that Six Sigma was more than quality control and statistics – ‘ultimately it drives leadership to be better by providing tools to think through tough issues’ – and rolled out their Six Sigma program in 1996 (Welch, 2001, p. 330). Examples of early Six Sigma success stories at GE, in both manufacturing and non-manufacturing situations, are reported by Welch (2001, pp. 333–334).

Perez-Wilson states that Motorola had looked for a catchy name to shake up the organization when introducing the concept of variation reduction and that in Six Sigma they found it. However, in spite of confusion over the different interpretations of sigma, in his opinion ‘It [Six Sigma] reflects a philosophy for pursuing perfection or excellence in everything an organization does. Six Sigma is probably the most successful program ever designed to produce change in an organization’ (Perez-Wilson, 1999, p. 195). A Six Sigma process, i.e. a process with a sigma quality level of 6, corresponds to 3.4 nonconformities per million opportunities – ‘That’s 99.99966 percent of perfection’ (Welch, 2001). Harry and Schroeder (2000) refer to a Six Sigma process as the Land of Oz and to the Six Sigma Breakthrough Strategy as the Yellow Brick Road leading there. The Six Sigma ‘roadmap’ is the subject of Section 1.3.

Antony (2010) and Montgomery and Woodall (2008) refer to the evolution of Six Sigma through three generations of implementations:

- Generation I – focus on elimination of defects and variation reduction, primarily in manufacturing. Spanned the period 1987–1994, with Motorola being a good exemplar.
- Generation II – in addition to the focus in the previous generation there was an emphasis on linking efforts to eliminate defects and reduce variation to efforts to improve product design and reduce costs. Spanned the period 1994–2000, with General Electric a prime exemplar.
- Generation III – since 2000 there has been an additional focus on value creation for both organizations and their stakeholders.

In a keynote presentation at the European Network for Business and Industrial Statistics conference in 2009, Tom Johnstone, CEO of global company SKF, referred to its employment of Six Sigma ‘to make it easier and attractive for our customers and suppliers to do business with us’. He referred to four dimensions – ‘Standard’ Six Sigma, Design for Six Sigma (DFSS), Lean Six Sigma and Six Sigma for Growth. He also referred to the integration of Six Sigma with other improvement initiatives as being a challenge faced by SKF (Johnstone, personal communication, 2010) This concurs with the comment by Montgomery and Woodall (2008) that they ‘expect Six Sigma to become somewhat less outwardly visible, while remaining an important initiative within companies’. These comments indicate that Six Sigma is still evolving. In the final section of their paper, on the future of Six Sigma, Montgomery and Woodall (2008) state:

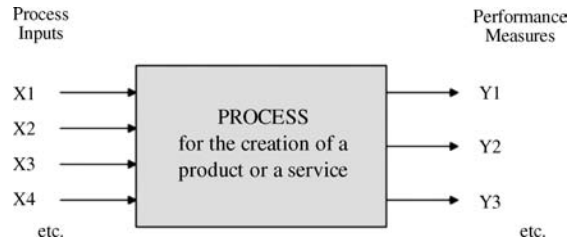
Six Sigma has become a widely used implementation vehicle for quality and business improvement. It is logical to ask about its future. Some have speculated that ‘Six Sigma’ is the ‘flavour of the month’ as management looks for the quick fix to crucial operational problems. However, since Six Sigma is over 20 years old and implementations are growing worldwide, it is difficult to believe that it is simply a management fad. In an ideal implementation, Six Sigma, DMAIC, DFSS and lean tools are used simultaneously in an organization to achieve high levels of process performance and significant business improvement.

Further evidence of the important role of Six Sigma is provided by the current development of two international standards, BS ISO 13053-1/2: *Quantitative methods in process improvement – Six Sigma – Part 1: DMAIC methodology* and *Part 2: Tools and techniques*.

### 1.3 The Six Sigma roadmap and DMAIC

The ideal roadmap for implementing Six Sigma within an organization is claimed to be as follows (Pande *et al.*, 2000).

1. Identify core processes and key customers.
2. Define customer requirements.



**Figure 1.3** Process model.

3. Measure current performance.
4. Prioritize, analyze, and implement improvements.
5. Expand and integrate the Six Sigma system.

A process or system may be represented by the schematic in Figure 1.3, where the  $X$ s represent inputs and the  $Y$ s represent performance measures on the process output. For the process of baking biscuits examples of inputs are flour supplier, oven temperature and cooking time. Some inputs are controllable by the baker, such as flour supplier and oven temperature. Others, such as the moisture content of the flour, are not. Examples of performance measures are the proportion of broken biscuits and the height of a stack of 16 biscuits – the number in a standard pack. Problems with the next process of packaging the biscuits may arise because stack height is off target or because there is too much variation in stack height.  $X$ s are referred to as inputs, independent variables, factors and key process input variables. Uncontrollable inputs are often designated noise factors.  $Y$ s are referred to as performance measures, dependent variables or responses. Where performance measures are of vital importance to the customer, the phrases key process output variables or critical to quality characteristics are employed.

The process improvement model at the heart of Six Sigma is often referred to as define–measure–analyse–improve–control (DMAIC). The first step is to define the problem or opportunity for improvement that is to be addressed. Writing in *Quality Progress*, a journal of the American Society for Quality (ASQ), Roger Hoerl, who at the time of writing is Manager of the Statistics Lab at General Electric Global Research, described the last four steps as shown in Box 1.1 (Hoerl, 1998). He also Hoerl (2001, p. 403) refers to DMAIC as ‘the glue which holds together the individual tools and facilitates solving real problems effectively’.

Some experts in Six Sigma apply the DMAIC improvement model both to process improvement projects and to process design/redesign projects. Others prefer to think in terms of DFSS and use define–measure–analyse–design–verify (DMADV) in the case of process design/redesign. Montgomery (2009) states that ‘the I in DMAIC may become DFSS’ – in other words, that improvement may only be possible through redesigning a process or creating a new one.

Six Sigma projects are normally selected as having potential in terms of major financial impact through facets of a business such as quality, costs, yield and capacity. Teams working on projects are typically led by employees designated ‘Black Belts’. Black Belts will have

*Measure* – Based on customer input, select the appropriate responses (the  $Y$ s) to be improved and ensure that they are quantifiable and can be accurately measured.

*Analyze* – Analyze the preliminary data to document current performance or baseline process capability. Begin identifying root causes of defects (the  $X$ s or independent variables) and their impact.

*Improve* – Determine how to intervene in the process to significantly reduce the defect levels. Several rounds of improvement may be required.

*Control* – Once the desired improvements have been made, put some type of system into place to ensure the improvements are sustained, even though additional Six Sigma resources may no longer be focused on the problem.

**Box 1.1** Description of key phases in applying Six Sigma methodology.

been awarded their titles after several weeks of intensive training, including statistical methods, and on completion of a successful Six Sigma project. A Six Sigma organization will invariably have a Six Sigma ‘Champion’ on the senior management team and may also have Master Black Belts. ‘Green Belts’ undergo less extensive training than Black Belts and may lead minor projects but normally assist on Black Belt led projects. Some organizations designate employees who have undergone basic Six Sigma training as ‘Yellow Belts’. External consultants are often used to train the first group of Black Belts within an organization. As the organization matures in terms of Six Sigma the Black Belts frequently undertake training of Green Belts and may devote 50–100% of their time to project activities. A discussion of curricula for the training of Black Belts is given by Hoerl (2001). He states that ‘the BB [Black Belt] role is intended to be a temporary assignment – typically two years’ (Hoerl, 2001, p. 394). Many organizations look to Black Belts to progress to senior roles; for example, at SKF it is envisaged that future company leaders will be former Black Belts (Johnstone, personal communication, 2010). Snee (2004) provides a Six Sigma project case study.

## 1.4 The role of statistical methods in Six Sigma

Measurement is fundamental to Six Sigma and measurement creates data. Many improvement initiatives lead to ‘before’ and ‘after’ data for  $Y$ s – data collected before and after process changes are implemented. In terms of the process model shown in Figure 1.3 the key to improvement is knowledge of how the  $X$ s influence the  $Y$ s, i.e. knowledge of the ‘formula’ linking the  $X$ s to the  $Y$ s, represented symbolically as  $\mathbf{Y} = f(\mathbf{X})$ . (The use of bold symbols indicates that invariably a process has associated with it a set of  $Y$ s and a set of  $X$ s so that  $\mathbf{Y}$  represents  $Y_1, Y_2, Y_3, \dots, Y_m$  and  $\mathbf{X}$  represents  $X_1, X_2, X_3, \dots, X_n$  in the case of a process with  $m$  performance measures and  $n$  inputs.) Statistics provides a series of tools to aid the search for such knowledge – tools such as design of experiments and regression.

The author recalls the late John Tukey stating, in a Royal Statistical Society presentation in Edinburgh in 1986, that ‘display is an obligation’ whenever one is dealing with data. Data display can be highly informative, so the topic will be emphasized. Tools from exploratory data