# Math Wise! 

Over 100 Hands-On Activities that Promote Real Math Understanding, Grades K-8

## Second Edition

Jim Overholt Laurie Kincheloe

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Over 100 Hands-On Activities that Promote Real Math Understanding, Grades K-8

## Second Edition

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## About This Resource

Math Wise! includes activities that will help each student gain full comprehension of basic mathematical concepts, including numbers and counting, computation, estimation, probability, data analysis, measurement, geometry, algebra, problem solving, and logical thinking. Students in today's math classrooms must be able to do more than achieve correct answers through computation; they need to understand basic concepts and experience a range of mathematical applications. Math Wise! is designed to help the teacher accomplish these learning objectives. It contains a wide variety of learning experiences that have been arranged according to difficulty level. Whenever possible, the activities are presented in either hands-on or visual formats.

## Concrete/Manipulative Activities

Especially when exploring "new" concepts, each student should work with hands-on materials. A number of the activities therefore include easily obtained manipulatives, such as straws, paper clips, sugar cubes, and beans. For example, a problem in the activity Paper Clip Division asks students to show 44 divided by 7 . One-to-one correspondence is used when one paper clip corresponds to the numeral 1. The result might appear as:


In Punchy Math, students use a paper hole punch, scrap paper, and a pencil to show $3 \times 7=$ $\qquad$ . The outcome, after folding, punching, looping, and labeling, shows 3 groups of 7 . If turned sideways, it can also show 7 groups of 3 , or $7 \times 3=$ $\qquad$ . Whereas the resulting punched holes are concrete, the looped segments provide a visual component that directly corresponds to the abstract number relationships involved.


Such manipulative activities provide a basis for true understanding of mathematical concepts. For this reason, each section contains a number of similar exercises.

## Visual/Pictorial Activities

For many learners, visual representations of mathematical problems are keys to the comprehension of these problems. Often visual representations involve 1-to-1 correspondence in connecting pictures with numbers. For example, in Cross-Line Multiplication, three horizontal lines represent the number 3 and five vertical lines represent the number 5. When the lines are crossed, the fifteen intersection points represent the answer to the problem $3 \times 5=$ $\qquad$ . The following figure illustrates this visual representation. Of course, turning the drawing sideways shows $5 \times 3=15$.

$3 \times 5=15$

$5 \times 3=15$

In Decimal Squares, another visual activity, students are provided with a sheet of Decimal Squares. Each decimal square is a 10 -unit by 10 -unit square divided into 100 square units. Each small square unit represents one hundredth of the decimal square, or .01 . Students are then asked to show the relationship between 0.6 and 0.21 . For example, in the problem 0.6 $\qquad$ 0.21 , students are required to fill in the blank with $>,<$, or $=$ to make the statement true. To find the answer, students are asked to shade in the Decimal Squares, as shown below.


## Abstract Procedures

A major goal of mathematics education is to help students eventually perform abstract mathematical procedures and understand the underlying concepts behind these procedures. When possible, mathematics teachers should not only instruct students in regard to mathematical mechanics but also enable them to gain a true understanding of the concepts involved.

In the activity Post-it Mental Math, one student has Post-it numerals placed on his or her back without being allowed to see them. The other group members, after viewing the numerals, give the student clues about the numerals. Using these clues, the Post-it wearer must do mental math to determine the numerals. In the situation that follows, the Post-it player has made a first guess based on one player's clues.


Block Four, which requires two players or two opposing teams, a numbered game board, and two paper clips, is another activity asking students to make abstract computations and draw upon their logicalthinking abilities. The first player places the paper clips on two numbers, and then performs the multiplication. The student then puts an $X$ on the square with the answer. The next player can only move one paper clip, leaving the other one alone. This player will then perform the multiplication and mark his or her square with an O . The boards below show two partially played games.

BLOCK FOUR
Multiplication Facts

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 9 | 10 | 12 | 14 |
| 15 | 16 | 18 | 2 | 21 | 24 |
| 25 | 27 | 2 | 30 | 32 | 35 |
| 36 | 40 | 42 | 45 | 48 | 49 |
| 54 | 56 | 63 | 64 | 72 | 81 |

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BLOCK FOUR
Multiplication of Fractions

| $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{21}{64}$ | $\frac{1}{36}$ | $\frac{1}{4}$ | $\frac{5}{32}$ | $\frac{5}{72}$ | $\frac{1}{54}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{1}{8}$ | $\frac{25}{48}$ | $\frac{1}{32}$ | $\frac{3}{16}$ | $\frac{1}{36}$ | $\frac{7}{24}$ | $\frac{1}{6}$ | $\frac{7}{72}$ |
| $\frac{1}{12}$ | $\frac{1}{2}$ | $\frac{3}{32}$ | $\frac{1}{9}$ | $\frac{5}{24}$ | $\frac{1}{16}$ | $\frac{7}{32}$ | $\frac{1}{16}$ |
| $\frac{5}{16}$ | $\frac{7}{48}$ | $\frac{1}{24}$ | $\frac{5}{48}$ | $\frac{3}{32}$ | $\frac{15}{64}$ | $\frac{1}{6}$ | $\frac{4}{9}$ |
| $\frac{9}{64}$ | $\frac{1}{8}$ | $\frac{2}{27}$ | $\frac{7}{12}$ | $\frac{1}{3}$ | $\frac{1}{27}$ | $\frac{1}{24}$ | $\frac{7}{16}$ |
| $\frac{15}{32}$ | $\frac{5}{12}$ | $\frac{9}{32}$ | $\frac{7}{64}$ | $\frac{1}{12}$ | $\frac{21}{31}$ | $\frac{1}{4}$ | $\frac{1}{64}$ |
| $\frac{1}{18}$ | $\frac{9}{16}$ | $\frac{25}{64}$ | $\frac{3}{8}$ | $\frac{1}{48}$ | $\frac{49}{64}$ | $\frac{3}{64}$ | $\frac{3}{16}$ |
| $\frac{1}{72}$ | $\frac{5}{24}$ | $\frac{1}{18}$ | $\frac{5}{64}$ | $\frac{1}{8}$ | $\frac{1}{18}$ | $\frac{35}{64}$ | $\frac{1}{12}$ |

(1) (2) $\frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{4}$

## A Final Note

Students will find the activities and investigations from this book informative, interesting, and fun. Most important, students will gain a better understanding of the mathematics they are expected to master. Math Wise! will prove to be a most valuable supplement to any mathematics program.

Jim Overholt Laurie Kincheloe

## About the Authors

James L. Overholt has an Ed.D. from the University of Wyoming, Laramie. He has been exploring the use of manipulative and visual materials for mathematics instruction since the 1960s. As an elementary and secondary school teacher in Minnesota and Wyoming, and later as a university professor, his investigations have taken him into both $\mathrm{K}-12$ classrooms and adult mathematics learning workshops. He is currently a professor of education at California State University, Chico.

Dr. Overholt regularly conducts mathematics education courses and workshops for pre-service and in-service teachers at the elementary and secondary levels. His earlier published books include Math Stories for Problem Solving Success, Second Edition, also published by JosseyBass/Wiley; Dr. Jim's Elementary Math Prescriptions; Math Problem Solving for Grades 4-8; Math Problem Solving for Beginners Through Grade 3; Outdoor Action Games for Elementary Children, and Indoor Action Games for Elementary Children.

Laurie Kincheloe has a B.A. in mathematics and an M.A. in mathematics education from California State University, Chico. She taught high school mathematics for twelve years and is presently teaching mathematics at Butte College in northern California. She has worked with $\mathrm{K}-12$ students, parents, and teachers as a family math coordinator and as a mentor for new teachers. She teaches concepts in mathematics to pre-service elementary teachers, and has coordinated service learning projects connecting high school and college students with elementary students through mathematics. She was co-coordinator of the Mathematics Project at California State University, Chico, and has conducted workshops on the teaching of mathematics for elementary and secondary teachers at numerous education conferences.

In addition to teaching at Butte College, Laurie has served as the developmental coordinator for the Mathematics Department, created a math-anxiety class designed to help apprehensive students be successful at math, and organized the annual Math Awareness Week. She has received the Faculty Member of the Year Award and the Service Learning Project Faculty Award.

A Special Acknowledgement:
James F. Lindsey, Ed.D. (University of California, Berkeley) served as an elementary teacher and principal for 25 years. He co-authored Math Stories for Problem Solving Success: Ready-to-Use Activities for Grades 6-12, First and Second Editions (Jossey-Bass/Wiley). When asked if he would help edit and proofread the new edition of Math Wise!, he remarked "I would be honored!'" From beginning to end, James was always ahead of expectations. He will be missed.

# Suggestions for Using Math Wise! 

The activities in this book provide a varied collection of interesting and understandable tasks from which students in kindergarten through the middle grades will benefit. Although many of these activities can be used in any order, it is advisable to designate tasks that are appropriate with regard to class size, students' stages of learning, or other considerations. For this reason, several features in this book are designed to help select appropriate activities.

- The Contents categorizes each activity in five ways:

1. Section ("Making Sense of Numbers," "Computation Connections," "Investigations and Problem Solving," and "Logical Thinking')
2. Descriptive Title (such as Everyday Things Numberbooks, Paper Clip Division, Peek Box Probability, and String Triangle Geometry)
3. Grade Level (K-2, 2-4, 4-6, and 6-8)
4. Activity Type (Concrete/Manipulative, Visual/Pictorial, and Abstract)
5. Learning Format (Total Group, Cooperative, and Independent)

- A Key for each activity notes the most appropriate grade levels, the preferred working arrangement, and the kinds of experiences in which learners will take part. For example, the following key to Silent Math indicates that

1. The activity is best suited for students in grades 4 through 8 .
2. The activity can be worked on by the whole class or by cooperative groups.
3. The students will work with visual diagrams and will perform abstract computations.

## Silent Math

## Grades 4－8

区 Total group activity
® Cooperative activity
$\square$ Independent activity
$\square$ Concrete／manipulative activity
区 Visual／pictorial activity
区 Abstract procedure
－Each activity begins with a Why Do It statement that details the specific mathematical concepts the students will be learning and practicing．
－The You Will Need statement specifies any supplies or equipment necessary for the activity．These items，such as paper clips，index cards，and straws，are easily obtained and free or inexpensive．
－The How To Do It section details what the teacher or other education professional must do to set up and carry out the activity． Suggestions are made as to the steps that should be taken for the activity to be successful．It also describes how the investigation works best as an independent activity，a cooperative project where students work in pairs or small groups，or a total group venture． This section will provide the general premise and content of the activity before the example are presented．
－The Examples illustrate how the activity might progress，and display typical outcomes．
－An Extensions section at the end of each activity contains more investigations that can be done using the same or similar procedures described in the activity．It often contains more sample questions or suggestions as to how to expand the mathematical concepts being studied．Teachers and students are encouraged to propose similar tasks of their own．
－Where appropriate，reproducible pages immediately follow the relevant activity．These pages include game boards，workmats，dot paper，playing cards，graph paper，and more．
－Students should be encouraged to record their methods and solu－ tions in a math journal or to keep a special file containing samples of their work．
－Solutions are also provided when appropriate．

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## Section One

## Making Sense of Numbers

The activities in this section introduce students to many number concepts and relationships, including 1-to-1 correspondence, basic number combinations, place value, mental math, fractions, large numbers, and decimals. Students will practice essential mathematical skills and develop conceptual understanding through hands-on investigations and games that make use of manipulative experiences, visual portrayals, or relevant abstract procedures.

A number of activities from other portions of this book can be used to extend and enhance students' comprehension of the concepts introduced in this section, such as Punchy Math (p. 104) and Beat the Calculator (p. 122) from Section Two; Peek Box Probability (p. 238) and Restaurant Menu Math (p. 235) from Section Three; and Duplicate Digit Logic (p. 408) from Section Four.

## Chapter 1

# Toothpick Storybooks 

## Grades K－3

Total group activity
® Cooperative activity
区 Independent activity
区 Concrete／manipulative activity
区 Visual／pictorial activity
区 Abstract procedure

## Why Do It：

Students will discover the concepts of 1－to－1 counting and number conservation，and will study basic computation rela－ tionships．

## You Will Need：

This activity requires several boxes of flat toothpicks，white and colored paper（pages approximately 6 by 9 inches work well），glue，and marking pens or crayons．

## How To Do lt：

1．Have younger students explore and share the different arrangements they can make with a given number of toothpicks．For example，students could arrange 4 toothpicks in a wide variety of different configu－ rations，all of which would still yield 4 toothpicks．
2．After exploring for a while，students should begin mak－ ing Toothpick Storybooks，starting by creating number pages．Students can write，for instance，the number 6 on a sheet of white paper and glue 6 toothpicks onto a piece of colored paper．（To avoid a sticky mess，students should dip only the ends of the toothpicks in the glue．）

When they are ready, the learners follow the same procedure for equations and the corresponding toothpick pictures. (Note: Students sometimes portray subtraction by pasting a small flap on the colored page that covers the number of toothpicks to be "taken away." Furthermore, they enjoy lifting the flap to rediscover the missing portion.)
3. When a number of toothpick diagrams have been finished, the pages can be stapled together into either individual or group Toothpick Storybooks. Ask each student to tell a number story about one of the diagrams in which he or she makes reference to both the toothpick figure and the written equation or number.

## Example:

Shown here are possible toothpick diagrams for 4, $3+5=\ldots$, and $7-2=$ $\qquad$ .


## Extensions:

1. Simple multiplication facts, and even longer problems, can be portrayed with toothpick diagrams. For $6 \times 3=$ $\qquad$ , the player might show ||| ||| ||| ||| ||| ||| = 18. Similarly, for $\overline{4 \times 23}=$ $\qquad$ , it is necessary to show 4 groups of 23 toothpicks to yield 92 .
2. Division can also be shown with toothpick diagrams. If the problem calls for the division of 110 into sets of 12 , the player would need to form as many groups of 12 as possible, also taking into account any remainder. (Note: The student might also complete such a problem using partitive division. See Paper Clip Division, p. 179.)

## Number Combination Noisy Boxes

## Grades K－3

Total group activity
区 Cooperative activity
区 Independent activity
® Concrete／manipulative activity
区 Visual／pictorial activity
区 Abstract procedure

## Why Do It：

This activity provides students with a visual and concrete aid that will help them understand basic number combinations and practice addition and subtraction．

## You Will Need：

Ten（or more）stationery or greet－ ing card boxes with clear plastic lids， approximately 50 marbles，and pieces of Styrofoam or sponge that can be trimmed to fit inside the boxes are
 required．

## How To Do It：

1．Construct Noisy Boxes for the numerals 0 through 9 （or beyond）．For each box，cut the foam to make a divider that will lie perpendicular to the bottom of the box．Glue the divider to the bottom of the box， ensuring that it is trimmed down such that the marbles
will pass over it when the top is on (see figure). Use a marking pen to write the numeral, such as 3, on the divider and to inscribe the appropriate number of dots on one outside edge of the box (- ) and the corresponding number word on another outside edge (three). Insert that same number of marbles into the box and tape on the clear plastic lid.
2. Allow the students to work with different Noisy Boxes. Instruct students to tip or shake a Noisy Box so that some or all of the marbles roll past the divider. Once this is done, the player is to record the outcome as an addition problem. The student should shake the same Noisy Box again and record a new outcome. For example, three marbles will yield outcomes such as $1+2,3+0$, $2+1$, or $0+3$. The activity continues in this manner until no further combinations are possible (see Example below).


## Example:

The recorded number combinations for the 7s Noisy Box should include the following:

| Addition |  | Subtraction |  |
| :---: | :---: | :---: | :---: |
| $4+3=7$ | $6+1=7$ | $7-4=3$ | $7-6=1$ |
| $3+4=7$ | $1+6=7$ | $7-3=4$ | $7-1=6$ |
| $5+2=7$ | $0+7=7$ | $7-5=2$ | $7-0=7$ |
| $2+5=7$ | $7+0=7$ | $7-2=5$ | $7-7=0$ |

## Extension:

If any player has difficulty on a visual level in utilizing a Noisy Box, have that student temporarily remove the plastic lid. Then he or she can touch and physically move the marbles from one side of the box to the other. Nearly all students will experience success as a result of such a tangible experience with number combinations.

## Chapter 3

# Everyday Things Numberbooks 

Grades K－4

区 Total group activity
区 Cooperative activity
ญ Independent activity
区 Concrete／manipulative activity
区 Visual／pictorial activity
区 Abstract procedure

## Why Do lt：

Students will discover that in their daily lives there are many things that come in numbered amounts，such as wheels on a bicycle．

## You Will Need：

Each student will require paper that can be stapled into booklets，pencils，scissors，and glue sticks or paste（if desired）．

## How To Do It：

1．As a group，discuss things in everyday life that are generally found as singles or 1 s ： 1 nose for each per－ son， 1 trunk per tree， 1 beak on a bird， 1 tail per cat， 1 －a－day multiple vitamins，and so on．Then provide each student with a sheet of paper and have every－ one write the number 2 at the top．Each participant should list as many things that come in 2 s as he or she can think of，such as 2 eyes，ears，hands，and legs for each person； 2 wings per bird，and so on．Do the same for 3 s ： 3 wheels on a tricycle； 3 sides for
any triangle; a 3-leaf clover, and so on. Students might also paste pictures representing numbered amounts on their pages. Have them complete a page (or more) for each number up to 10 or larger, and then discuss their ideas. You may want to construct large class lists for each number. This activity can continue for several days, and may be assigned as homework.
2. At first some numbers seem unusable, but wait and you will be delighted with students' suggestions. For instance, 7 can be illustrated by 7 -UP ${ }^{\circledR}$, and 8 depicted by 8 sides on a stop sign. Students will often continue to make suggestions, even after the activity has ended!

## Example:

The following is a partial Numberbook listing for the number 4.


## Extensions:

Ask more advanced students to consider the following problems:

1. What items can commonly be found in 25 s, 50 s, 100 s, or any other number you or students might come up with? Is there any number for which an example cannot be found?
2. Find examples for fractional numbers. If there are 12 sections in an orange, 1 of those sections is $1 / 12$ of the orange; 3 of those sections are $3 / 12$ or $1 / 4$ of the orange.

# Chapter 4 

# Under the Bowl 

## Grades K－3

$\square$ Total group activity
区 Cooperative activity
区 Independent activity
区 Concrete／manipulative activity
® Visual／pictorial activity
区 Abstract procedure

## Why Do It：

Under the Bowl provides students with a visual and concrete aid that will help them understand basic number combina－ tions and practice addition and subtraction．

## You Will Need：

A bowl or small box lid and small objects（such as beans， blocks，or bread tags）are required for each player．

## How To Do lt：

Allow students a brief period to explore their bowls and objects．Have students begin the activity with small numbers of items：students with 3 beans，for example，might be told to put 2 beans under the bowl and place the other on top of it．Then they should say aloud to a partner or together as a class，＂One bean on top and two beans underneath make three beans altogether．＂Once students understand the activity，ask them to keep a written record of their work；for 3 beans，as noted above，they should record $1+2=3$（after they have had instruction on four fact families，they should also record $2+1=3,3-2=1$ ，and $3-1=2$ ）．Although
initially students should use only a few objects, they might go on to use as many as 20,30 , or even 100 items.


## Example:

The players shown above are working with 7 beans. Thus far they have recorded the four fact family for 1 bean on top of the bowl and 6 beans under it. They are now beginning to record their findings for 2 beans on top. Next they might put 3 beans on top and record. (Note: Should a student become confused about a number combination, he or she may count the objects on top and then lift the bowl to either visually or physically count the objects underneath. This usually helps clarify the problem.)

## Extensions:

1. When older students are working as partners, an interesting variation has one student making a combination and the other trying to figure out what it is. For example, the first student might put 3 beans on top of the bowl and some others under it. He or she then states, "I have 11 beans altogether. How many beans are under the bowl, and what equations can you write to represent this problem?'' The second student should respond that there are 8 beans under the bowl, yielding the equations $3+8=11,8+3=11,11-8=3$, and $11-3=8$.
2. You can also extend this activity to introduce algebra concepts to students. For example, after instruction a student presents the problem shown in Extension 1, with the equation $n+3=11$. Explain to students that using a letter to represent a missing number is a basic concept in algebra.
