

Volume 2  
of An Analytical Commentary on  
the *Philosophical Investigations*

# Wittgenstein: Rules, Grammar and Necessity

Essays and Exegesis of §§185–242



G. P. Baker & P. M. S. Hacker

Fellows of St John's College · Oxford

Second, extensively revised edition

by

P. M. S. Hacker



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P. M. S. Hacker

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For Anne and Sylvia





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A version of the essay 'Private linguists and "private linguists" – Robinson Crusoe sails again' was presented at a conference organized by Nuño Venturinha at the Universidade Nova de Lisboa in May, 2008, and is to be published in the volume he has edited, entitled *Wittgenstein after His Nachlass* (Palgrave Macmillan, Basingstoke, 2009). Parts of the essay 'Grammar and necessity' were presented at seminars at the University of Bologna in April/May 2009 and at the 32nd International Wittgenstein Symposium at Kirchberg in August 2009.

P. M. S. H.

Thoughts reduced to paper are generally nothing more than the footprints of a man walking in the sand. It is true that we see the path he has taken; but to know what he saw on the way, we must use our own eyes.

Schopenhauer

## Introduction to Volume 2

The first edition of this book was written between 1981 and 1984. Gordon Baker and I had not originally intended to dedicate a whole volume to the fifty-seven remarks that run from *Philosophical Investigations* §185 to §242. But we found that the text was exceedingly difficult to penetrate. The interpretative controversies about these remarks were extensive and deep. The amount of manuscript material on rules, following rules, practices and techniques was large. The ratio of directly related *Nachlass* notes and typescripts to published text is high. For, as we noted, Wittgenstein went over this ground again and again, criss-cross in all directions, repeatedly redrafting remarks, adding, pruning and polishing. Much of this material is invaluable, making his intentions clear and resolving disputes about the interpretation of the final text. We thought it a fundamental part of our enterprise to lay this documentation before the reader. The manuscript material is indispensable for understanding Wittgenstein's ideas, and it provides the background against which our exposition of his ideas must be judged. So we resolved to write a volume dedicated to Wittgenstein's views on grammatical rules, on accord with a rule, on following rules, on internal relations, and on the nature of logical, grammatical and mathematical necessity.

Because the themes raised in these fifty-seven remarks of the *Investigations* are so densely interwoven, the essays in this volume are more closely integrated as parts of a single logical nexus than those in Volume 1, Part I. The clarification of what precisely Wittgenstein meant by 'a rule of grammar', what the relation is between a rule and the acts that accord with it, what it is to follow a rule, and whether, and in what sense, one can follow a rule privately or 'privately' are a sequentially related array of enquiries that are essential to understanding Wittgenstein's thought both upon the philosophy of mathematics and upon the philosophy of language. Precisely because the *Frühfassung* continued into an early draft of the *Remarks on the Foundations of Mathematics*, Part 1, rather than into what we now know as 'the private language arguments', we thought it essential to explain how two such diverse trees as the philosophy of mathematics and explanation of the nature of necessity, on the one hand, and the private language arguments, on the other, could be grafted onto the same stock. And we also thought it necessary to take some steps to explain the direction of Wittgenstein's thought in the *Remarks*. Hence the long concluding essay on 'Grammar and necessity'. For it is not possible to understand

Wittgenstein's thought in general without some grasp of his conception of the nature of the necessary truths of logic, mathematics and grammar (metaphysics), even though this theme is muted in the *Investigations*.

While we were working on this volume of the Commentary, Saul Kripke published his lecture on Wittgenstein on following rules, which was subsequently expanded into a small book, *Wittgenstein on Rules and Private Language* (Blackwell, Oxford, 1982). We were convinced, on the basis of our research on the *Nachlass*, that Kripke's (and Robert Fogelin's prior) interpretation of what they both called the 'paradox' of rule-following was mistaken. We were even more doubtful about the sceptical Humean solution to the alleged problem, which they attributed to Wittgenstein, and of the assertability-conditional semantic theory that Kripke, following Dummett, ascribed to Wittgenstein. We confronted those views explicitly in a small volume of three essays on the subject, *Scepticism, Rules and Language* (Blackwell, Oxford, 1984), and implicitly in this volume, especially in the essays 'Accord with a rule' and 'Following rules, mastery of techniques and practices' and in the exegesis of §§198–202. We showed that they had no adequate basis in Wittgenstein's writings. Indeed, we argued that they conflict with much that Wittgenstein had written and with the point and purpose of his discussions.

In the quarter of a century since we published the first edition of this volume, there has been much discussion of Kripke's interpretation, some criticizing and others defending it. Some philosophers, such as Norman Malcolm, prescind from Kripke's sceptical interpretation of the remarks on following rules, but supported the so-called community view according to which there can be rules only if they are shared by a community of rule-followers. These continuing debates merited fresh scrutiny. New *Nachlass* materials were discovered, shedding further light on the issues. The Bergen electronic edition of Wittgenstein's *Nachlass* was published, with an invaluable search engine. Puzzlement and bafflement about Wittgenstein's writings on the philosophy of mathematics continued, still largely guided by Michael Dummett's misinterpretations of Wittgenstein's *Remarks on the Foundations of Mathematics* in his 1959 review of the book and in his later writings on the subject. All this warranted an extensively revised edition that would take advantage of the new materials and of the *Nachlass* search engine, and examine the issues afresh.

In 2001 Gordon Baker and I decided to produce a second edition of Volume 1 of this Commentary, *Wittgenstein: Understanding and Meaning*, in order to correct errors that we had made twenty-five years earlier, to bring it up to date with the continuing research on Wittgenstein's texts, to make use of the electronic edition of the *Nachlass* and its search engine, and to elaborate the new ideas we had had in the course of a quarter of a century's further reflections. Before we had even begun, however, Gordon fell ill with cancer, and died in 2002. Working therefore alone, I rewrote large parts of the first edition of *Wittgenstein: Understanding and Meaning*, compressing the old text and adding a great deal of new material, as well as two new essays, completely redrafting

about half of the old essays, and comprehensively rewriting the exegesis. The second edition was published in two volumes, one of essays and the other of exegesis, in 2005. Having completed that task, I turned in 2007 to examine Volume 2 of the Commentary, and decided that this too needed redrafting and supplementing with new material.

In this second edition I have redrafted much of the exegesis in the light of numerous debates on the text over the last twenty-five years. I have, as in the revisions to Volume 1, benefited greatly from Professor Eike von Savigny's methodical criticisms in his *Wittgensteins 'Philosophische Untersuchungen': Ein Kommentar für Leser*, 2nd edn (Klosterman, Frankfurt am Main, 1994). I have also been able to make use of the Bergen electronic edition to hunt down passages Gordon and I had missed as we searched our way through the 20,000 pages of *Nachlass* in the 1980s. Hence the tables of correlation are much expanded, and will enable scholars to trace each remark to its sources. The result of this new research is, I hope, an exhaustive survey of the relevant materials that bear on the text, which will enable readers to judge for themselves whether the interpretation offered is faithful to the printed text and supported by the relevant manuscript materials. I should emphasize that I bear sole responsibility for any new views advanced in this second edition.

The essays have been rewritten, sometimes extensively. One point that I have tried to bring out, which was not previously made adequately clear, is the point and purpose of Wittgenstein's lengthy investigations into rules and following rules, and their structural role within the *Philosophical Investigations*. We had endeavoured to clarify why this issue arises out of the conception of meaning as use, on the one hand, and the notion of understanding, in particular understanding something at a stroke, on the other. But no less pertinent is the need to elucidate the nexus between internal relations and the idea that following a rule is a practice. For internal relations are the fruits of a normative *practice* – a rule-governed regularity of *action*. And, against the backdrop of the stream of human life, it is they that determine what is to be *called*, for example, describing, calculating, inferring, and so forth. These ideas are at the heart of Wittgenstein's normative conception of mathematical propositions and laws of inference and, indeed, it was for that purpose that they were originally crafted. For the book was originally intended to proceed from §189 into what we now know as Part I of the *Remarks on the Foundations of Mathematics*. So it is important to set the discussion of following a rule in *both* contexts.

One new essay has been added. 'Private linguists and "private linguists" – Robinson Crusoe sails again' is designed to settle the debate between the 'community view' and its 'individualist' adversaries once and for all as far as Wittgensteinian exegesis is concerned.

By far the most important modification to the essays is the substantial expansion of 'Grammar and necessity'. I have rewritten this essay, compressing the old text and adding much new material. My intention was to produce an overview of Wittgenstein's conception of logical, grammatical and mathematical



necessity and also a prolegomenon to his later (post-1936) philosophy of mathematics. A new section on mathematical proof has been added, with an explanation of Wittgenstein's remarks on decision that is at odds with the views commonly ascribed to him. I have also elaborated the section on the relationship between Wittgenstein's philosophy of mathematics and logic and that of the Vienna Circle. This was rewritten in the light of Gordon Baker's essay on this subject in *Wittgenstein, Frege and the Vienna Circle* (Blackwell, Oxford, 1988).

The exegesis follows the same model as in Volume 1, Part II, save that in this volume it is dispersed between the essays. It has been thoroughly revised. The number of German quotations has been reduced, since the text of Wittgenstein's manuscripts is now readily available in the Bergen electronic edition of the *Nachlass*. The text of the *Philosophische Untersuchungen* used is the 4th edition. The English text of the *Investigations* is also the 4th edition with its considerably modified translation.

P. M. S. Hacker  
St John's College, Oxford  
March, 2009



# Abbreviations

## 1. Wittgenstein's published works

The following abbreviations, listed in alphabetical order, are used to refer to Wittgenstein's published works.

- BB        *The Blue and Brown Books* (Blackwell, Oxford, 1958).
- BIB       Occasionally used to refer to the *Blue Book*.
- BrB       Occasionally used to refer to the *Brown Book*.
- BT        *The Big Typescript*, tr. C. G. Luckhardt and M. A. E. Aue (Blackwell, Oxford, 2004).
- C        *On Certainty*, ed. G. E. M. Anscombe and G. H. von Wright, tr. D. Paul and G. E. M. Anscombe (Blackwell, Oxford, 1969).
- CE        'Cause and Effect: Intuitive Awareness', repr. in *Ludwig Wittgenstein: Philosophical Occasions 1912–1951*, ed. J. Klagge and A. Nordmann (Hackett, Indianapolis and Cambridge, 1993), pp. 370–426.
- CL        *Cambridge Letters*, ed. Brian McGuinness and G. H. von Wright (Blackwell, Oxford, 1995).
- CV        *Culture and Value*, ed. G. H. von Wright in collaboration with H. Nyman, tr. P. Winch (Blackwell, Oxford, 1980).
- EPB       *Eine Philosophische Betrachtung*, ed. R. Rhees, in *Ludwig Wittgenstein: Schriften 5* (Suhrkamp, Frankfurt, 1970).
- GB        'Remarks on Frazer's "Golden Bough"', tr. J. Beversluis, repr. in *Ludwig Wittgenstein: Philosophical Occasions 1912–1951*, ed. J. Klagge and A. Nordmann (Hackett, Indianapolis and Cambridge, 1993), pp. 118–55.
- LPE       'Wittgenstein's Notes for Lectures on "Private Experience" and "Sense Data"', ed. R. Rhees, repr. in *Ludwig Wittgenstein: Philosophical Occasions 1912–1951*, ed. J. Klagge and A. Nordmann (Hackett, Indianapolis and Cambridge, 1993), pp. 202–88.
- LW I     *Last Writings on the Philosophy of Psychology*, vol. I, ed. G. H. von Wright and H. Nyman, tr. C. G. Luckhardt and M. A. E. Aue (Blackwell, Oxford, 1982).
- NB        *Notebooks 1914–16*, ed. G. H. von Wright and G. E. M. Anscombe, tr. G. E. M. Anscombe 2nd edn (Blackwell, Oxford, 1979).
- PG        *Philosophical Grammar*, ed. R. Rhees, tr. A. J. P. Kenny (Blackwell, Oxford, 1974).

- PI *Philosophical Investigations*, revised 4th edn edited by P. M. S. Hacker and Joachim Schulte, tr. G. E. M. Anscombe, P. M. S. Hacker and Joachim Schulte (Wiley-Blackwell, Oxford, 2009).
- PO *Ludwig Wittgenstein: Philosophical Occasions 1912–1951*, ed. J. Klagge and A. Nordmann (Hackett, Indianapolis and Cambridge, 1993).
- PPF *Philosophy of Psychology – A Fragment* published in *Philosophical Investigations*, revised 4th edn edited by P. M. S. Hacker and Joachim Schulte, tr. G. E. M. Anscombe, P. M. S. Hacker and Joachim Schulte (Wiley-Blackwell, Oxford, 2009).
- PR *Philosophical Remarks*, ed. R. Rhees, tr. R. Hargreaves and R. White (Blackwell, Oxford, 1975).
- PTLP *Proto-Tractatus – An Early Version of Tractatus Logico-Philosophicus*, ed. B. F. McGuinness, T. Nyberg and G. H. von Wright, tr. D. F. Pears and B. F. McGuinness (Routledge and Kegan Paul, London, 1971).
- RC *Remarks on Colour*, ed. G. E. M. Anscombe and G. H. von Wright, tr. L. L. McAlister and M. Schättle (Blackwell, Oxford, [1977]).
- RFM *Remarks on the Foundations of Mathematics*, ed. G. H. von Wright, R. Rhees, G. E. M. Anscombe, rev. edn (Blackwell, Oxford, 1978).
- RLF ‘Some Remarks on Logical Form’, *Proceedings of the Aristotelian Society*, suppl. vol. 9 (1929), pp. 162–71.
- RPP I *Remarks on the Philosophy of Psychology*, vol. I, ed. G. E. M. Anscombe and G. H. von Wright, tr. G. E. M. Anscombe (Blackwell, Oxford, 1980).
- RPP II *Remarks on the Philosophy of Psychology*, vol. II, ed. G. H. von Wright and H. Nyman, tr. C. G. Luckhardt and M. A. E. Aue (Blackwell, Oxford, 1980).
- TLP *Tractatus Logico-Philosophicus*, tr. D. F. Pears and B. F. McGuinness (Routledge and Kegan Paul, London, 1961).
- Z *Zettel*, ed. G. E. M. Anscombe and G. H. von Wright, tr. G. E. M. Anscombe (Blackwell, Oxford, 1967).

Reference style: all references to *Philosophical Investigations* are to sections (e.g. PI §1), except those to boxed notes on various pages. Reference to these pages is given by two numbers, the first referring to the page of the first and second editions, the second to the fourth edition. References to *Philosophy of Psychology – a Fragment* (previously known as *Philosophical Investigations Part II*) are to numbered sections, and to page numbers in the first two editions (e.g. PPF §1/p. 174). References to other printed works are either to numbered remarks (TLP) or to sections signified ‘§’ (Z, RPP, LW); in all other cases references are to pages (e.g. LFM 21 = LFM, page 21) or to numbered letters (CL); references to *The Big Typescript* are to the original pagination of the typescript as given in the Bergen electronic edition of *Wittgenstein’s Nachlass* (Oxford University Press, Oxford, 2000) and in the published translation edited by Aue and Luckhardt.

## 2. Derivative primary sources and Waismann's publications

- AWL     *Wittgenstein's Lectures, Cambridge 1932–35, from the Notes of Alice Ambrose and Margaret MacDonald*, ed. Alice Ambrose (Blackwell, Oxford, 1979).
- IMT     *Introduction to Mathematical Thinking*, F. Waismann, tr. T. J. Benac (Hafner, London, 1951).
- LA     *Lectures and Conversations on Aesthetics, Psychology and Religious Beliefs*, ed. C. Barrett (Blackwell, Oxford, 1970).
- LFM     *Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge 1939*, ed. C. Diamond (Harvester Press, Sussex, 1976).
- LPM     *Lectures on the Philosophy of Mathematics*, F. Waismann, ed. with an introduction by W. Grassl (Rodopi, Amsterdam, 1982).
- LPP     *Wittgenstein's Lectures on Philosophical Psychology 1946–47*, notes by P. T. Geach, K. J. Shah and A. C. Jackson, ed. P. T. Geach (Harvester Wheatsheaf, Hemel Hempstead, 1988).
- LSD     'The Language of Sense Data and Private Experience', notes taken by R. Rhees of Wittgenstein's lectures, 1936, *Philosophical Investigations* 7 (1984), pp. 1–45, 101–40.
- LWL     *Wittgenstein's Lectures, Cambridge 1930–32, from the Notes of John King and Desmond Lee*, ed. Desmond Lee (Blackwell, Oxford, 1980).
- M     G. E. Moore's notes entitled 'Wittgenstein's Lectures in 1930–33', repr. in *Ludwig Wittgenstein: Philosophical Occasions 1912–1951*, ed. J. Klagge and A. Nordmann (Hackett, Indianapolis and Cambridge, 1993), pp. 46–114.
- PLP     *The Principles of Linguistic Philosophy*, F. Waismann, ed. R. Harre (Macmillan and St Martin's Press, London and New York, 1965).
- RR     *Discussions of Wittgenstein*, by R. Rhees (Routledge and Kegan Paul, London, 1970).
- VoW     *The Voices of Wittgenstein*, transcribed and edited by Gordon Baker, tr. Gordon Baker, Michael Mackert, John Connolly and Vasilis Politis (Routledge, London, 2003).
- WWK     *Ludwig Wittgenstein und der Wiener Kreis*, shorthand notes recorded by F. Waismann, ed. B. F. McGuinness (Blackwell, Oxford, 1967). The English translation, *Wittgenstein and the Vienna Circle* (Blackwell, Oxford, 1979), matches the pagination of the original edition.

## 3. *Nachlass*

All references to other material cited in the von Wright catalogue (G. H. von Wright, *Wittgenstein* [Blackwell, Oxford, 1982], pp. 35ff.) are by MS or TS number followed by page number ('r' indicating recto, 'v' indicating verso) or section number '§', as it appears in the Bergen electronic edition of *Wittgenstein's Nachlass*.

In the case of the first manuscript draft of the *Investigations*, MS 142 (the *Urfassung*), references are to Wittgenstein's section number ('§'), save in the case of references to pp. 77f., which are redrafts of PI §§1–2 and to pp. 78–91,

which Wittgenstein crossed out and redrafted on pp. 91ff., subsequently assigning them section numbers in the redrafts alone.

### Manuscripts

MSS 105–22 refer to the eighteen large manuscript volumes written between 2 February 1929 and 1944. These were numbered by Wittgenstein as Vols I–XVIII. In the first edition of this Commentary they were referred to by volume number, followed by page number (e.g. ‘Vol. XII, 271’). Since then it has become customary to refer to them by von Wright number alone. Here they are referred to on their first occurrence in a discussion by their von Wright number, followed by volume number in parenthesis, followed by page number as paginated in the Bergen edition (e.g. ‘MS 116 (Vol. XII), 271’). In the subsequent occurrence of a reference to the same volume in the same discussion, the volume number is dropped.

‘MS 114 (Vol. X), Um.’ refers to Wittgenstein’s pagination of the *Umarbeitung* (reworking) of the *Big Typescript* in MS 114. The *Umarbeitung* begins on folio 31v of MS 114 (Vol. X), and is consecutively paginated 1–228.

### Typescripts

B I *Bemerkungen I* (TS 228), 1945–6, 185 pp. All references are to numbered sections (§).

All other typescripts are referred to as ‘TS’, followed by the von Wright number and pagination as in the Bergen edition.

The successive drafts of the *Investigations* are referred to as follows:

TS 220 is the typescript of the ‘Early Draft’ (*Frühfassung* (FF)) of the *Investigations*, referred to in the first edition of this Commentary as ‘PPI’ (‘Proto-Philosophical Investigations’), dictated from MS 142 (the *Urfassung* (UF)).

TS 226<sub>R</sub> is Rhees’s pre-war translation of TS 220 §§1–116, referred to in the 1st edn of this Commentary as PPI(R).

TS 227a and 227b are the two surviving typescripts of the *Investigations* (the copy from which the text was printed having been lost).

TS 238 is a reworking of TS 220, §§96–116, with renumberings, deletions, corrections and additions in Wittgenstein’s hand, referred to in the 1st edn of this Commentary as PPI (A).

TS 239 is a reworking of TS 220 (*Bearbeitete Frühfassung*).

ZF is the reconstructed ‘Intermediate Draft’ (*Zwischenfassung*) of the *Investigations*, previously known as ‘The Intermediate Version’, and referred to in the 1st edn of this Commentary as PPI(I).

In transcriptions from the *Nachlass* I have followed Wittgenstein’s convention of enclosing alternative draftings within double slashes ‘//’.

#### 4. Abbreviations for the other volumes of *An Analytical Commentary on the Philosophical Investigations*

G. P. Baker and P. M. S. Hacker, *Wittgenstein: Understanding and Meaning, Volume 1 of An Analytical Commentary on the Philosophical Investigations*, 2nd edn, extensively revised by P. M. S. Hacker, Part I – Essays, Part II – Exegesis §§1–184 (Blackwell, Oxford, 2005).

P. M. S. Hacker, *Wittgenstein: Meaning and Mind, Volume 3 of an Analytical Commentary on the Philosophical Investigations* (Blackwell, Oxford and Cambridge, Mass., 1990).

P. M. S. Hacker, *Wittgenstein: Mind and Will, Volume 4 of an Analytical Commentary on the Philosophical Investigations* (Blackwell, Oxford and Cambridge, Mass., 1996).

All references to these are of the form ‘Volume’, followed by the volume number and the quoted title of an essay in the designated volume (and, in the case of Volume 1, to Part I). References to the exegesis are flagged ‘Exg.’, followed by section number prefixed with ‘§’ or page number (in the case of the boxed remarks).

#### 5. Abbreviations for works by Frege

BLA i *The Basic Laws of Arithmetic*, vol. i (1893); references to the preface by roman numeral indicating original page number, all other references by section number (§).

BLA ii *The Basic Laws of Arithmetic*, vol. ii (1903); all references by section number (§).

CN *Conceptual Notation and Related Articles*, tr. and ed. T. W. Bynum (Clarendon Press, Oxford, 1972). References to sections (§) are to ‘Conceptual Notation’.

FA *The Foundations of Arithmetic*, tr. J. L. Austin, 2nd edition (Blackwell, Oxford, 1959).

PW *Posthumous Writings*, ed. H. Hermes, F. Kambartel and F. Kaulbach, tr. P. Long and R. White (Blackwell, Oxford, 1979).

#### 6. Abbreviations for works by Russell

PM *Principia Mathematica*, vol. I (with A. N. Whitehead), 2nd edn (Cambridge University Press, Cambridge, 1927).

PrM *The Principles of Mathematics*, 2nd edn (rev.) (Allen and Unwin, London, 1937).



ANALYTICAL  
COMMENTARY







## Two fruits upon one tree

### 1. *The continuation of the Early Draft into philosophy of mathematics*

The constructional history of the *Investigations* (see Volume 1, Part II, ‘The history of the composition of the *Philosophical Investigations*’) is a matter of interest not only to chroniclers of the history of ideas. It bears directly on features of Wittgenstein’s thought. For it raises a general question about his later philosophy, namely: how can two such disparate fruits as his philosophy of logic and mathematics and examination of the nature of necessity, on the one hand, and his investigations into the possibility of a private language, knowledge of other minds, the inner and the outer, the nature of thinking, imagination, consciousness and the self, on the other, grow from the same trunk of *Investigations* §§1–189? For the *Frühfassung* (the Early Draft) of 1937–8 consisted of TS 220 (corresponding roughly to PI §§1–189(a)) and TS 221 (corresponding roughly to RFM I). It was only in 1944 that Wittgenstein decided to drop the logico-mathematical continuation from the book, and to allow the material on following rules to evolve not into a discussion of the nature of necessity (as in TS 221), but rather into the private language arguments and their sequel up to §421 (which was the terminus of the *Zwischenfassung* (the Intermediate Draft) of 1944). This incorporated a mere eight pages from TS 221, corresponding to PI §§189–97, which was then followed by new material composed in 1944. Furthermore, in the *Spätfassung* (the Late Draft) of 1946–7 Wittgenstein added yet further material (including §§422–693) incorporating *inter alia* discussions of topics in the philosophy of psychology (such as intentionality, the will, intention), which for the most part bear, directly or indirectly, on the general theme of the nature of linguistic representation.

What motivated this metamorphosis? What did he conceive to be the relationship between philosophy of mathematics and philosophy of psychology? What can we learn from this realignment about his tactics, strategy and grand-strategy? Clearly many issues can be probed. Our ambitions here are limited to a textual and a methodological question. The textual question concerns the integration of the remarks and the structure of the chain of reasoning in the published text in the light of the divergent earlier continuation. The methodological question concerns parallelisms between Wittgenstein’s philosophy of mathematics and his philosophy of psychology.

In comparison with standard works on philosophy of mathematics, Part I of the *Remarks on the Foundations of Mathematics* is unusual. Philosophers of mathematics might expect discussions of Russell's paradox, of how to construct real numbers from the rationals, of the acceptability of indirect proofs, or of the cogency of mathematical induction. Here we find none of this. Instead Wittgenstein investigated the concepts of proof and inference, compared calculations with experiments, juxtaposed logical with legal compulsion, demythologized logical, mathematical and metaphysical necessity, and so on. His points are illustrated with very elementary examples, such as  $25 \times 25 = 625$ , or simple diagrammatic proofs of equations. Only the innumerate would have difficulty following these.

Apart from what the editors conjecture to be projected appendices on Cantor's theory of infinity, Gödel's incompleteness proof, Russell's logic and logicist definitions of natural numbers, Wittgenstein avoided the subject called 'the foundations of mathematics' (an amalgam of formal logic, number theory and real analysis). He incorporated no mathematical or metamathematical results into his work and he criticized attempts to extract philosophical theses from such proofs. This has evoked hostility among philosophers of mathematics and logicians, who have often misinterpreted this *methodological* disagreement as a manifestation of ignorance of sophisticated mathematics or even of philistinism. He abstained from frontal attacks on standard positions in philosophy of mathematics, neither allying himself with logicists, intuitionists or formalists nor lining up against them under some other banner (strict finitism or constructivism). But in the course of his investigations into mathematical concepts he made devastating criticisms, almost *en passant*, of each of the familiar triad. His reluctance to locate himself in some available pigeonhole has not diminished others' enthusiasm for doing so. Whatever positive conception he had, must, it has sometimes been thought, either be a synthesis of the three<sup>1</sup> or be a purified version of one of them.<sup>2</sup>

Disregarding the themes of the three so-called appendices, Wittgenstein explored five interrelated topics in the text of the original continuation of the Early Draft beyond what is now PI §189:

(i) *Inference* Inferring or drawing a conclusion is not a mental process or act, but a transformation of expressions according to paradigms. It is not answerable to something external but is a movement within grammar. Rules of inference do not flow from the meanings of the logical constants, but rather are constitutive of these meanings. To explain that *fa* follows from  $(x)fx$  is to give a partial explanation of what the universal quantifier means, and to explain

<sup>1</sup> In the Manifesto of the Vienna Circle (1929), the signatories (Hahn, Neurath and Carnap) assert that 'Some hold that the three views are not so far apart as it seems. They surmise that essential features of all three will come closer in the course of future development and probably, using the far-reaching ideas of Wittgenstein, will be united in the ultimate solution.' (See *The Scientific Conception of the World: The Vienna Circle* (Dordrecht, Holland, 1973), p. 13.)

<sup>2</sup> For example, intuitionism purged of psychologism.

the nature of an inference is to teach someone the technique of inferring (drawing conclusions, reasoning, thinking).

(ii) *Proof and calculation* Proofs in mathematics are commonly conflated with proofs (inferences) outside mathematics (and logic). But the ‘diplomatic’ or ‘imperial’ role of a mathematical proof or proposition, the role that gives point to the body of mathematics, is to supply a paradigm for the transformation of empirical statements, i.e. to establish a pattern of inference. The nature of mathematics is obscured by the fact that we express mathematical results in the form of declarative sentences; but we might carry on mathematics without thinking that we were dealing with propositions at all (RFM 93, 117). We construe mathematical proofs as demonstrations of propositions from other propositions, but this too is inessential, since a proof may consist of a diagram or geometrical construction to which the concepts of premises, conclusions and inference are inapplicable (MS 161, 6, see Exg. §144). It is also misleading in suggesting that a mathematical proposition is fully intelligible independently of its proof; but a conclusion is best conceived as the end surface of a proof-body (AWL 10). A proof establishes internal relations; it connects concepts and thereby contributes to the determination of their identity. Proofs and calculations are thus radically unlike experiments (empirical verifications).

(iii) *Essence and convention* Precisely because a mathematical proof establishes internal relations, it also *creates essence* (RFM 50). For in fixing novel internal relations, a proof extends the grammar of number or space. By extending grammar, essences are modified and created – normative (conceptual) connections are determined. The mathematician is an inventor, not a discoverer (RFM 99), for he determines new relations among concepts. To accept a proof is to accept a new rule, a new *convention* – which the proof has woven into the tapestry of mathematics. Indeed, talk about essences is no more than noting conventions (RFM 65). For ‘internal properties’ are marks of *concepts* (RFM 64). They seem adamant, unassailable, independent of the vagaries of fortune – but their unassailability is that of *shadows* (RFM 74). Statements of essences seem to describe the language-independent, *de re* necessities of things, but they are actually determinations of forms of description and of forms of transformations of descriptions of things.

(iv) *Logical compulsion* Logical inference seems inexorable. Anybody who believes the premises of a valid argument seems to be logically *compelled* to believe the conclusion. Misleading pictures surround the ‘hardness of the logical “must”’, for example, that the conclusion is somehow *already contained* in its premises, that ‘the mathematical proof drives us along willy-nilly’ until we arrive at the conclusion, or that if we wish to ‘remain faithful’ to the concepts we are employing, then logic compels us to accept the theorem as proven – we have no more choice in the matter than does the appropriately programmed computer. But, Wittgenstein argues, these pictures are no more than mythological representations of necessity. They are to be countered by clarifying the

concept of inference. We are instructed in the techniques of inferring and compelled by our teachers and peers to adhere to them. A specification of what conclusion to draw from given premises is an intrinsic feature of the technique of inference. Hence we are not *forced* by logic to draw a conclusion – no matter what conclusion we draw, logic will not seize us by the throat! That such-and-such a conclusion *follows* (or even: ‘follows necessarily’) is not a form of compulsion. Rather, we are constrained in our *judgements* about what is to be *called* ‘a correct inference’. Wittgenstein’s account of inference does not derogate from the inexorability of logic. It merely eliminates misconceptions of it.

(v) *The natural history of mankind – regularity and the role of agreement* The practices of inference, proof, calculation and reasoning presuppose a ramifying network of regularities in nature and human behaviour. We typically respond similarly to patterns of instruction in arithmetical techniques. We normally agree in our judgements when applying such techniques. Mathematicians rarely quarrel over whether something is a proof. Certain patterns or resemblances are memorable for us, whereas we have ‘blind spots’ for other possibilities or similarities. Such regularities of agreement and response are not parts of (do not define) our concept of proof or inference. Rather, these regularities are part of the framework within which we exercise these concepts. Without them our language-games would lose their point. Hence, in clarification of our concepts it is useful to note our established patterns of action and speech, our form of life or culture, and also other contrasting ones, whether real or imaginary.

Even this brief survey discloses affinities between the early mathematical continuations of the Early Draft and the published text. Both give central positions to clarifying the concepts of a rule, of correctness according to a rule, and of following a rule. Wittgenstein argued that arithmetical equations and geometrical theorems should be viewed not as descriptions of numbers and shapes, but as rules the general point and purpose of which lies in the transformation of empirical propositions concerning, for example, magnitudes, quantities, distances, velocities and spatial relationships of things. The immediate rationale of the initial continuation of the Early Draft is the elucidation of the concepts of mathematical proposition, mathematical necessity and the nature of proof for the purpose of removing prevalent philosophical confusions. This task is interwoven with the project of illuminating central normative<sup>3</sup> concepts characteristic not only of mathematics but of language use in general. Only the latter material has direct parallels in the published text.

At the cost of a certain amount of rearrangement and perhaps some supplementation of the early text, it seems that Wittgenstein could have separated out the remarks on rules, accord with a rule and following rules, and then treated these as a preface to his discussion of logical inference and mathematical proof. The result might have been something like the published text of

<sup>3</sup> By ‘normative’ we mean merely ‘pertaining to a norm (rule)’.

§§189–242, followed by material on philosophy of mathematics rather than on the ‘private language’ and its sequel. Apparently something roughly like this idea occurred to him, probably in 1943 or 1944. Under the heading ‘Plan’ he outlined this programme:

How can the rule determine what I have to do?

To follow a rule presupposes agreement.

It is essential to the phenomenon of language that we do not dispute about certain things.

How can agreement be a condition of language? . . . Were agreement lacking, i.e. were we to be unable to bring our expressions into agreement, then the phenomena of communication and language would disappear too.

In what does the inexorability of mathematics consist?

The way goes from what is not inexorable to inexorability. The word ‘oben’ has four sounds.

Is a mathematical proof an experiment? (MS 165, 30ff.)

Here the envisaged argument has the same general contour as §§189–242, but it then diverges into a discussion of mathematics in order to remove the objection that describing agreement in judgements as a presupposition of language is inconsistent with the inexorability of mathematical propositions and proofs. Apparently Wittgenstein contemplated deploying remarks on inference, proof and logical compulsion in order to show in detail that the need for agreement in judgement does not abolish logic though it seems to do so, i.e. that acknowledging this framework condition for language is not incompatible with recognizing the hardness of the logical ‘must’ (in so far as this is intelligible; cf. ‘Agreement in definitions, judgements and forms of life’ and ‘Grammar and necessity’.) Consequently we might view the continuation of the Early Draft as an important complement to the final version. In elaborating the implications of agreement, it removes potential misunderstandings.

## 2. *Hidden isomorphism*

It is a moot question how a uniform foundation – a variant of *Investigations* §§1–189 – can underlie each of two such divergent extensions. How is it possible that a single chain of reasoning should lead smoothly into remarks on mathematics and logic, or alternatively into the private language arguments and discussions of psychological concepts? One response, natural in the light of the foregoing observations, would be that one must beware of exaggerating the divergence and of overlooking the shared features. As just noted, in both texts Wittgenstein discussed the internal relation of a rule to its applications, the autonomy of grammar, and the role of agreement as a framework-condition for following rules. Should one not view the final discussion of following rules as part of the shared nucleus of the two extensions? Then the

early extension can be seen as exploring one main objection to the conceptual role assigned to agreement, viz. that logic and mathematics (or more generally, logical necessity) would be undermined. And the later extension can be viewed as examining a second fundamental objection, viz. that a language is conceivable (a 'private language') independently of even the possibility of agreement. Accordingly the two divergent continuations complement each other, and each graft fits perfectly on to the same trunk.

An alternative response, antithetical to the first, would beware of exaggerating the shared features. The pivot on which both continuations turn is a set of remarks about following a rule; but is the pivot identical? There seems to be a shift of emphasis in the discussion of following a rule between the *Remarks on the Foundations of Mathematics*, Part I and the *Investigations*. Wittgenstein's focus of attention moved towards a sharper concentration upon the framework-conditions of rule-governed activities. *Investigations* §§185–242 stands at the culmination of this development, incorporating and moving onwards from the scrutiny of the internal relations between a rule and its application that began in *The Big Typescript*. One must not take for granted that there is no substantial evolution here, that the examination of following a rule in Part I of the *Remarks* has not been deepened and enriched in the *Investigations*. A fortiori one may not argue that the earlier extension of the Early Draft and *Investigations* §§243ff. are extensions of a homogeneous set of ideas dubbed 'Wittgenstein's rule-following considerations'.

Both responses are justified. We shall not attempt to adjudicate the question of what degree of continuity obtains and how extensive a change occurred. For our purposes, the important point to stress is that the text of the Early Draft was written after much reflection on philosophy of mathematics and on such topics in the philosophy of psychology that are pertinent to an overview of language and linguistic meaning. It was informed by a unified conception of philosophy and philosophical methods. It was expressly designed to highlight sources of philosophical confusion rampant both in philosophy of mathematics and in those parts of philosophy of psychology that are pertinent to the nature of thought, language and linguistic representation. Under the rubric of the Augustinian conception of language, the Early Draft drew together and surveyed a wide range of points that Wittgenstein had already made in earlier writings. The two divergent continuations fit on to this common foundation because it was crafted to support either, not as a result of personal idiosyncrasy, but for deeper philosophical reasons. We shall justify this claim by examining briefly two of the main ingredients of the Augustinian conception and by elucidating the manner in which it prevents philosophical understanding.

(i) *Descriptions* It is part of the Augustinian conception of language to take it for granted that the fundamental role of sentences, certainly of declarative sentences, is to describe something (see Volume 1, Part I: 'The Augustinian conception of language', §1 and §2(e)). Philosophers often *begin* their reflections from this presupposition. 'Eight is greater than five' is presumed to